ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics & Statistics B

Unit MBP4

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Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– x EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

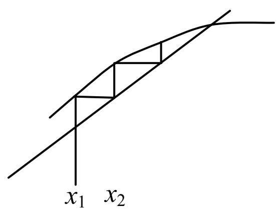
MC – x	deducted x marks for miscopy
MR – x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question Number and part	Solution	Marks	Total	Comments
1(a)	$A = 500$	B1		
	$10k = \ln\left(\frac{750}{A}\right)$	M1		Substitute $P = 750$, $t = 10$ and attempt to find k using ln
	$k = \frac{1}{10}\ln\left(\frac{3}{2}\right) \approx 0.0405$	A1	3	Exact value or at least 1 SF 0.0405465…
(b)	$kt = \ln\left(\frac{1500}{A}\right)$	M1		
	$t = 10\frac{\ln 3}{\ln 1.5} \approx 27.1$	A1	2	Accept 27.095 (11291…) Condone more SF rounding to 27.1 if correct working
	Total		**5**	
2(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 8\left(x^3+1\right)^{-1} - 24x^3\left(x^3+1\right)^{-2}$	M1		Product (must have –ve powers)/quotient rule attempt $\frac{8-16x^3}{(x^3+1)^2}$
		A1		Correct unsimplified
	When $x = 1, \frac{\mathrm{d}y}{\mathrm{d}x} = -2$	A1	3	cso; all working must be correct
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$	M1		Any correct version – stated or used
	$= -2 \times 0.8 = -1.6$	A1√	2	ft from their part(a) answer
(ii)	Negative sign $\Rightarrow$ y decreasing Losing height, going down etc	E1 √	1	ft positive value $\Rightarrow y$ increasing NOT speed/rate of change etc decreasing
	Total		**6**	
3(a)	$p(-1) = 2 \times -3 \times -5$	M1		Or full long division as far as remainder
	$= 30$	A1	2	
(b)(i)	$\frac{A}{x+3} + \frac{B}{x-2} + \frac{C}{x-4}$	M1		Comparing coeffs or substituting values
		A1		First term correct
	$A = 2,\quad B = -7,\quad C = 5$	A1	3	All terms correct
(ii)	$A\ln(x+3) + B\ln(x-2) + C\ln(x-4)$	M1		Integration involving ln
		A1 √		ft their A, B, C
	$[2\ln 9 - 7\ln 4 + 5\ln 2]$ $-[2\ln 8 - 7\ln 3 + 5\ln 1]$	m1		Sub'n of limits 6 and 5 (condone slip)
	$= 4\ln 3 - 14\ln 2 + 5\ln 2 - 6\ln 2 + 7\ln 3$	B1		2 correct simplifications of p ln2, q ln3 $\ln 9 = 2\ln 3$, $\ln 4 = 2\ln 2$, $\ln 8 = 3\ln 2$
	$= 11\ln 3 - 15\ln 2$	A1	5	
	Total		**10**	

Question Number and part	Solution	Marks	Total	Comments
4(a)(i)	$x^2 + y^2 - 10x - 6y + \frac{111}{4}$	M1		Attempt at completing square or one coordinate correct (generous)
	Centre (5, 3)	A1	2	
(ii)	$r^2 = 25 + 9 - \frac{111}{4} = \frac{25}{4}$	M1		3 numbers - condone sign error
	$r = \frac{5}{2}$	A1	2	**oe**
(b)(i)	$\frac{\lvert 5\times 3 - 3\times 4 - 16\rvert}{\sqrt{(3^2 + 4^2)}}$	M1		Strict on formula use but ft their centre
	$= \frac{13}{5}$	A1	2	Must be positive
(ii)	2.6 > radius $\Rightarrow$ does NOT intersect	E1 √	1	ft deduction from their distance & radius
(iii)	$m_1 = 2$; $m_2 = \frac{3}{4}$	B1		Both gradients given
	$\tan\theta = \left\lvert \frac{2 - \frac{3}{4}}{1 + \frac{3}{2}} \right\rvert = \frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$	M1		Use of angle between lines formula or equivalent method
		A1	3	**ag** $(\Rightarrow \theta = \tan^{-1}\frac{1}{2}$ not needed)
	Total		**10**	
5(a)	$x_2 = 3.742$	B1		Condone more than 3 dps if rounding to these values.
	$x_3 = 3.968$	B1		
	$x_4 = 3.996$	B1	3	cso
(b)(i)	$x_{n+1} \to L$; $x_n \to L \Rightarrow L = \sqrt{(L+12)}$	M1		
	$\Rightarrow L^2 = L + 12 \Rightarrow L^2 - L - 12 = 0$	A1	2	
(ii)	$(L-4)(L+3) = 0$	M1		Factor or formula attempt
	$L = 4, L = -3$			
	$x_n > 0, \forall n \Rightarrow L = 4$	A1	2	Rejecting negative value and answer = 4 Award M1,A0 if value 4 is given with no evidence of discarding the negative value
(c)	Vertical line to curve first then horizontal line to $y = x$	M1		
	Staircase convergence shown (at least 2 horizontal sections)	A1	2	
	Total		**9**	

Question Number and part	Solution	Marks	Total	Comments
6(a)	$\cos x\cos\frac{5\pi}{6}-\sin x\sin\frac{5\pi}{6}=\sin x$	M1		
	$\cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}$	B1		
	$\sin\frac{5\pi}{6}=\frac{1}{2}$	B1		$-\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=\sin x$
	$\sqrt{3}\cos x+3\sin x=0$			
	$\Rightarrow\sqrt{3}\sin x+\cos x=0$	A1	4	Be convinced $\sqrt{3}$ not fudged **ag**
(b)	$\tan x=-\frac{1}{\sqrt{3}}$	M1		$\tan x=\ldots$, $\sin^2 x=\ldots$, $\cos^2 x=\ldots$
	$x=\frac{5\pi}{6},\ \frac{11\pi}{6}$	A1		Condone 150° or 2.61799…rads
		A1	3	must both be in radians and in terms of π
	Total		**7**	
7(a)(i)	$\frac{dy}{dx}=2\sec^2 2x$	M1 A1	2	$A\sec^2 kx$ correct
(ii)	$x=\frac{\pi}{6}\Rightarrow\frac{dy}{dx}=8$	B1		Differentiation must be correct
	Tangent equation is $y-\sqrt{3}=8\left(x-\frac{\pi}{6}\right)$	B1 √	2	ft gradient (any form for line)
(b)	$\left[\frac{1}{2}\tan 2x-\ln\sec 2x\right]$	M1 A1 A1	3	Integration : $A\tan 2x$ or $B\ln\sec 2x$ One term correct All terms correct
(c)(i)	$\alpha=\frac{\pi}{8}$	B1	1	
(ii)	$(\pi)\int(\tan 2x-1)^2\,dx$	M1		$V=\pi\int_0^\alpha\left(\tan^2 2x+1-2\tan 2x\right)dx$
	sight of $\sec^2 2x=1+\tan^2 2x$ Shown to equal	B1		
	$V=\pi\int_0^\alpha\left(\sec^2 2x-2\tan 2x\right)dx$	A1	3	**ag**
(iii)	$\frac{1}{2}\tan\frac{\pi}{4}-\ln\sec\frac{\pi}{4}$ or $\frac{1}{2}\tan 2\alpha-\ln\sec 2\alpha$ $\Rightarrow V=\pi\left(\frac{1}{2}-\ln\sqrt{2}\right)$	M1		Limits used on their answer to (b) Accept in terms of α Condone missing π for M1
	$=\frac{\pi}{2}(1-\ln 2)$	A1	2	**ag** proved convincingly $\ln\sqrt{2}=\frac{1}{2}\ln 2$ etc
	Total		**13**	
	TOTAL		**60**	