General Certificate of Education
January 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBP3**
Unit Pure 3

Monday 19 January 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- an insert for use in Question 5 (enclosed);
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach this insert to your answer book.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The roots of the quadratic equation $x^2 + 2x + 3 = 0$ are α and β.

(a) Without solving the equation:

(i) write down the value of $\alpha + \beta$ and the value of $\alpha\beta$; *(2 marks)*

(ii) show that $\alpha^3 + \beta^3 = 10$; *(3 marks)*

(iii) find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$. *(2 marks)*

(b) Determine a quadratic equation with integer coefficients which has roots

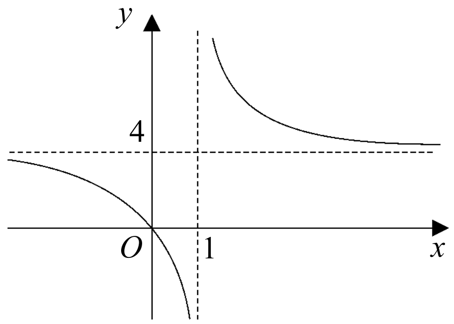
$$\frac{1}{\alpha^3} \quad \text{and} \quad \frac{1}{\beta^3}$$ *(3 marks)*

2 (a) Sketch the curve with equation $y^2 = 8x$. *(2 marks)*

(b) Write down the equation of the curve obtained when the curve $y^2 = 8x$ is reflected in the line $y = x$. *(2 marks)*

(c) Describe a geometrical transformation that maps the curve $y^2 = 8x$ onto the curve with equation $y^2 = 8x - 16$. *(2 marks)*

3 The graph of $y = \mathrm{f}(x)$ is sketched below.
The asymptotes have equations $x = 1$ and $y = 4$.



(a) Given that $\mathrm{f}(x) = \frac{ax}{x - b}$, use the sketch to find the values of a and b. *(2 marks)*

(b) Sketch the graph of $y^2 = \mathrm{f}(x)$ and state the equations of its asymptotes. *(5 marks)*

4 The matrix $\boldsymbol{A}$ is $\begin{bmatrix} 4 & k \\ 3 & 6 \end{bmatrix}$.

(a) Find the determinant of $\boldsymbol{A}$. *(1 mark)*

(b) Find the value of k for which the inverse of $\boldsymbol{A}$ does not exist. *(2 marks)*

(c) The transformation $\mathbf{T}$ is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

(i) A triangle has area 5 square units.

Find the area of its image under $\mathbf{T}$ in the case when $k = 8$. *(1 mark)*

(ii) Find the possible values of k in order that, under $\mathbf{T}$, a triangle with area 5 square units would be mapped onto a triangle with area 15 square units. *(3 marks)*

5 [An insert is provided for use in answering this question.]

The variables Q and x satisfy a relationship of the form $Q = ax^b$, where a and b are constants.

Measurements of Q for given values of x gave the following results.

x	0.4	0.5	0.6	0.7	0.8
Q	1.72	3.02	4.74	6.98	9.73

(a) Express $\ln Q$ in terms of $\ln a$, b and $\ln x$. *(1 mark)*

(b) (i) Complete the table on the insert and plot $\ln Q$ against $\ln x$ on the axes provided. *(3 marks)*

(ii) Draw a suitable straight line to illustrate the relationship between the data. *(1 mark)*

(c) Use your line to estimate:

(i) the value of Q when $x = 0.54$, giving your answer to two significant figures; *(2 marks)*

(ii) the values of a and b, giving your answers to two significant figures. *(4 marks)*

6 (a) Find the value of the following, giving each answer in the form $a + b\mathrm{i}$, where a and b are integers.

(i) $(2 + 3\mathrm{i})^2$ *(2 marks)*

(ii) $(2 + 3\mathrm{i})^4$ *(2 marks)*

(b) (i) Given that $2 + 3\mathrm{i}$ is a root of the equation

$$z^4 + 40z + k = 0$$

find the value of the real constant k. *(2 marks)*

(ii) Write down another root of the equation $z^4 + 40z + k = 0$. *(1 mark)*

7 Part of the Cayley table for the set $S = \{2, 4, 6, 8, 10, 12\}$ under multiplication modulo 14 is given below.

	2	**4**	**6**	**8**	**10**	**12**
2	4	8	12	2	6	10
4	8	2	10	4	12	6
6	12	10				
8	2	4				
10	6	12				
12	10	6				

For example, $4 \times 10 = 40 \equiv 12 \pmod{14}$

(a) (i) Copy and complete the Cayley table. *(4 marks)*

(ii) Explain why the set S is closed under multiplication modulo 14. *(1 mark)*

(iii) State the identity element. *(1 mark)*

(iv) Find the inverse of 12. *(1 mark)*

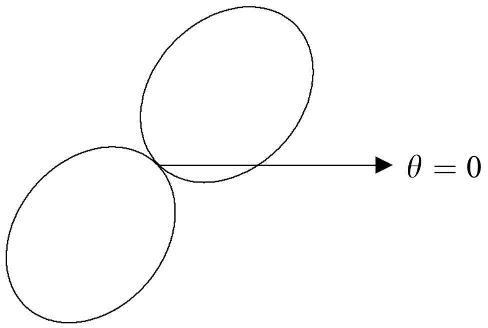
(b) Find a solution of the congruence $10x \equiv 4 \pmod{14}$. *(1 mark)*

(c) Given that x is a member of S, find the possible values of x for which $x^2 + 12 \equiv 0 \pmod{14}$. *(2 marks)*

8 The curve C sketched below has equation

$$r = 1 + \sin 2\theta$$

where $[r, \theta]$ are polar coordinates and $-\pi < \theta \leqslant \pi$.



(a) Find the greatest value of r and the corresponding values of θ. *(3 marks)*

(b) Find the exact values of θ for which $r = 0$. *(4 marks)*

(c) Given that the polar equation of C can also be written in the form

$$r = 1 + 2\sin\theta\cos\theta$$

find a cartesian equation for C.

You need not simplify your answer. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

9 (a) (i) Given that

$$f(r) = (r-1)r(r+1)(r+2)$$

show that

$$f(r+1) - f(r) = kr(r+1)(r+2)$$

stating the value of the constant k. *(2 marks)*

(ii) Use the method of differences to find $\sum_{r=1}^{n} r(r+1)(r+2)$, giving your answer in factorised form. *(3 marks)*

(b) (i) Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$ *(6 marks)*

(ii) Hence find $\sum_{r=1}^{\infty} \frac{2}{r(r+1)(r+2)}$. *(1 mark)*

END OF QUESTIONS