ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics & Statistics B

Unit MBP3

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Registered address AQA, Devas Street, Manchester M15 6EX. *Dr Michael Cresswell Director General*

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– *x* EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

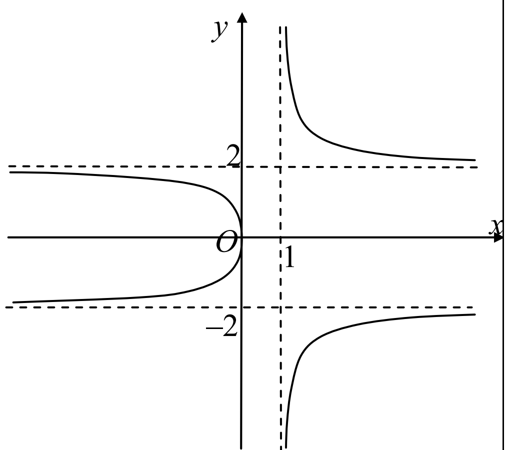
Abbreviations used in marking

MC – *x*	deducted x marks for miscopy
MR – *x*	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Question number and part	Solution	Marks	Total	Comments
1(a)(i)	$\alpha+\beta=-2\,,\quad \alpha\beta=3$	B1 B1	2	
(ii)	$(\alpha+\beta)^3-3\,\alpha\beta(\alpha+\beta)$	M1 A1		Or $(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ & $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
	$\Rightarrow \alpha^3+\beta^3=10$	A1	3	**ag**
(iii)	$\dfrac{\alpha^3+\beta^3}{(\alpha\beta)^3}=\dfrac{10}{27}$	M1 A1	2	
(b)	New product of roots = $\dfrac{1}{(\alpha\beta)^3}=\dfrac{1}{27}$	B1		
	x^2 – [cand's (a) (iii)] x + [cand's product]	M1		
	$\Rightarrow \quad 27x^2-10\,x+1=0$	A1√	3	ft Must have integer coefficients and be an equation
	Total		**10**	
2(a)	⊂ - shaped parabola	M1		
	Vertex at O, good sketch, symmetry obvious	A1	2	*Essentially* all correct
(b)	$x^2=8\,y$ or equivalent	M1 A1	2	M1 for general idea
(c)	Translation; by vector $\begin{bmatrix}2\\0\end{bmatrix}$	M1 A1	2	**sc**: B1 for correct description without "translation"
	Total		**6**	
3(a)	$a=4$ and $b=1$	B1 B1	2	
(b)	Asymptotes $x=1\,,\ y=2\,,\ y=-2$	B1 B1		One correct; second correct
	Graph: Correct for $y>0$	B1		
	Symmetry in x-axis	B1		
	All correct	B1	5	Or B1 for each correct region
				E.g. 4/5 for all correct graph but with asymptotes $x=1, y=\pm 4$
	Total		**7**	

Question number and part	Solution	Marks	Total	Comments
4(a)	$24 - 3k$	B1	1	
(b)	Det = 0 $\Rightarrow$ $k = 8$	M1√ A1√	2	ft (a)
(c)(i)	Area = 0	B1√	1	ft 5 × cand's Det with $k = 8$
(ii)	Det = 3 and / or − 3 $\Rightarrow$ $k = 7$ $\Rightarrow$ $k = 9$	M1 A1√ A1	3	ft cand's "$24 - 3k = 3$" cao
	Total		**7**	
5(a)	$\ln Q = \ln a + b \ln x$	B1	1	
(b)(i)	$\ln x$: –0.92 –0.69 –0.51 –0.36 –0.22 $\ln Q$: 0.54 1.11 1.56 1.94 2.28 Points plotted on graph provided	B1 B1 B1	3	Most correct At most one error Reasonably accurately
(ii)	"Good" line of best fit drawn	B1	1	
(c)(i)	$\ln Q = 1.29 - 1.30$ $\Rightarrow$ $Q \approx 3.6 - 3.7$	M1 A1	2	
(ii)	Method for finding gradient: $b = 2.5$ Reading off y-intercept: $\ln a \approx 2.8$ $a = 16 - 17$	M1 A1 M1 A1	4	± 0.1 Give M marks for simultaneous equations approach
	Total		**11**	
6(a)(i)	$-5 + 12\text{ i}$	M1 A1	2	
(ii)	Squaring their answer to (i) or use of the binomial theorem: $-119 - 120\text{ i}$	M1 A1√	2	ft
(b)(i)	Subst^{g}. their z^4, $z = 2 + 3\text{ i}$ into equation $(-119 - 120\text{ i}) + 40(2 + 3\text{ i}) + k = 0$ $\Rightarrow$ $k = 39$	M1 A1	2	cao
(ii)	$2 - 3\text{ i}$	B1	1	Or $z = -1, -3$
	Total		**7**	
7(a)(i)	8 6 4 2 6 8 10 12 4 10 2 8 2 12 8 4	B1 B1 B1 B1	4	One for each correct row/column
(ii)	Only elements of S appear in the Cayley table	E1	1	Or equivalent statements
(iii)	The identity is 8	B1	1	
(iv)	$12^{-1} = 10$	B1	1	
(b)	$x \equiv 6 \pmod{14}$ but allow $x = 6$	B1	1	
(c)	$x = 4$ and $x = 10$	B1 B1	2	**sc** B1 for $x^2 \equiv 2 \pmod{14}$ only
	Total		**10**	

Question number and part	Solution	Marks	Total	Comments
8(a)	$r_{max} = 2$ when $\theta = \frac{1}{4}\pi$ and $\theta = -\frac{3}{4}\pi$	B1 B1 B1	3	
(b)	$\sin 2\theta = -1 \Rightarrow 2\theta = \frac{3}{2}\pi, \ldots$	M1 A1		
	giving $\theta = \frac{3}{4}\pi, \quad \theta = -\frac{1}{4}\pi$	A1 A1	4	Penalise degrees max. once; ignore correct out-of-range answers
(c)	Use of $r = \sqrt{x^2 + y^2}$	B1		
	Use of either $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$	M1		
	$\sqrt{x^2 + y^2} = 1 + \frac{2xy}{x^2 + y^2}$	A1	3	Any correct form at earliest stage
	Total		**10**	
9(a)(i)	Attempt at $f(r+1) - f(r)$ $= r(r+1)(r+2)(r+3)$ $- (r-1)r(r+1)(r+2)$ $= r(r+1)(r+2)\{r + 3 - r + 1\}$ $= 4r(r+1)(r+2)$	M1 A1	2	i.e $k = 4$
(ii)	$\sum r(r+1)(r+2) = \frac{1}{4}\sum\{f(r+1) - f(r)\}$	M1√		ft k
	$= \frac{1}{4}\{f(n+1) - f(1)\}$	m1		
	$= \frac{1}{4}n(n+1)(n+2)(n+3)$	A1	3	
(b)	For $n = 1$, LHS = RHS = $\frac{1}{3}$	B1		
	Adding next term to at least the RHS	M1		
	Correct $(k+1)^{th}$ term used: $\frac{2}{(k+1)(k+2)(k+3)}$	B1		
	RHS = $\frac{1}{2} - \left\{\frac{(k+3) - 2}{(k+1)(k+2)(k+3)}\right\}$	M1		With correct sign
	$= \frac{1}{2} - \frac{1}{(k+2)(k+3)}$	A1		
	Clear induction hypothesis somewhere	E1	6	Or full explanation at end
(c)	$S = \frac{1}{2}$	B1	1	
	Total		**12**	
	TOTAL		**80**	