General Certificate of Education
January 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS **MBP2**
(SPECIFICATION B)
Unit Pure 2

Friday 16 January 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

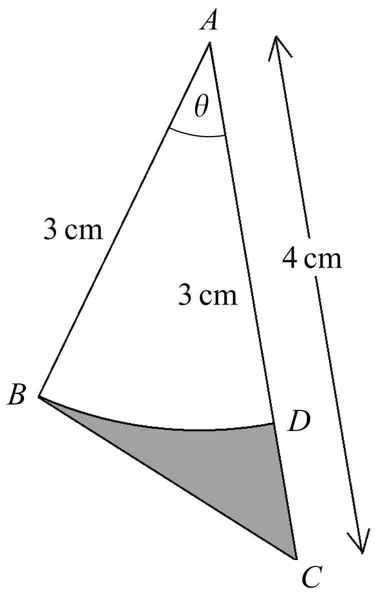
Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Show that $\int_0^1 (e^{2x} + 4)\,dx = \frac{1}{2}(e^2 + 7)$. *(4 marks)*

2 The diagram shows a triangle ABC with $AB = 3$ cm, $AC = 4$ cm and angle $BAC = \theta$ radians.



The point D lies on AC such that $AD = 3$ cm, and ABD is a sector of a circle with centre A and radius 3 cm.

(a) Write down, in terms of θ:

(i) the area of the sector ABD; *(2 marks)*

(ii) the area of triangle ABC. *(2 marks)*

(b) Show that, for small values of θ, the area of the shaded region, bounded by the lines BC and DC and the arc BD, is approximately 1.5θ square centimetres. *(2 marks)*

3 On one particular day the volume, $V\,\text{m}^3$, of liquid in a tank changes with time, t hours after midnight, according to the formula

$$V = 8 + 6e^{-\frac{1}{12}t}$$

(a) Find the volume of liquid in the tank when $t = 0$. *(1 mark)*

(b) Find the rate of change, in m^3 per hour, of the volume of liquid in the tank when $t = 12$, interpreting the sign of your answer. *(3 marks)*

(c) Find, to the nearest minute, the time when the volume of liquid in the tank is $11\,\text{m}^3$. *(4 marks)*

4 Solve the equation

$$\tan x = -\sqrt{3}$$

in the interval $0 < x < 2\pi$, leaving your answers in terms of π in their simplest form. *(4 marks)*

5 (a) Sketch the graph of $y = |2x - 4|$. Indicate the coordinates of the points where the graph meets the coordinate axes. *(4 marks)*

(b) (i) The line $y = x$ intersects the graph of $y = |2x - 4|$ at two points P and Q. Find the x-coordinates of the points P and Q. *(3 marks)*

(ii) Hence solve the inequality $|2x - 4| > x$. *(2 marks)*

(c) The graph of $y = |2x - 4| + k$ touches the line $y = x$ at only one point. Find the value of the constant k. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ▶

6 A geometric series has common ratio r. The first term of the series is 2.

(a) (i) Write down the first four terms of the series in terms of r. *(1 mark)*

(ii) The sum of the first four terms of the series is $\frac{15}{4}$. Show that

$$8r^3 + 8r^2 + 8r - 7 = 0$$ *(3 marks)*

(b) (i) Use the factor theorem to show that $(2r - 1)$ is a factor of the polynomial $\mathrm{p}(r) = 8r^3 + 8r^2 + 8r - 7$. *(2 marks)*

(ii) Hence factorise $8r^3 + 8r^2 + 8r - 7$ as the product of a linear and a quadratic factor. *(2 marks)*

(iii) Hence show that the equation $\mathrm{p}(r) = 0$ has only one real solution. *(2 marks)*

(c) Find the sum to infinity of the geometric series. *(3 marks)*

7 A curve has equation

$$y = \frac{1}{4}x^3 - 6\ln x + 1, \quad x > 0$$

(a) (i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. *(2 marks)*

(ii) Hence show that the gradient of the curve at the point where $x = \frac{2}{3}$ is $-8\frac{2}{3}$. *(2 marks)*

(b) (i) Given that the curve has just one stationary point, find the x-coordinate of this stationary point. *(3 marks)*

(ii) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(2 marks)*

(iii) Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4.5$ at the stationary point and hence state the nature of the stationary point. *(2 marks)*

(c) P and Q are two points on the curve $y = \frac{1}{4}x^3 - 6\ln x + 1$. The x-coordinate of P is 4 and the x-coordinate of Q is 8.

Find the gradient of the chord PQ in the form $a + b\ln 2$, where a and b are constants to be found. *(3 marks)*

END OF QUESTIONS