

General Certificate of Education
January 2004
Advanced Subsidiary Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Pure 1**

MBP1

Thursday 8 January 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

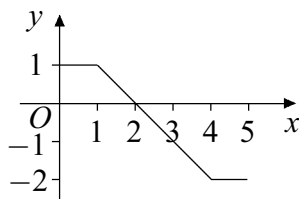
Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1** The first term of an arithmetic series is 7. The tenth term is 43.
- (a) Find the common difference. *(2 marks)*
 - (b) Find the sum of the first fifty terms of the series. *(3 marks)*
 - (c) The k th term has a value greater than 1000.
 - (i) Show that $4k > 997$. *(2 marks)*
 - (ii) Find the least possible value of k . *(1 mark)*
- 2** The point A has coordinates $(-1, 2)$ and the line AB is parallel to the line joining the points $(7, 4)$ and $(3, 8)$.
- (a)
 - (i) Find the gradient of AB . *(1 mark)*
 - (ii) Show that the line AB has equation $x + y = 1$. *(1 mark)*
 - (b) The point C has coordinates $(5, 0)$ and the line BC is perpendicular to the line with equation $x + 3y = 7$.
 - (i) Find the gradient of BC . *(2 marks)*
 - (ii) Find an equation for the line BC . *(1 mark)*
 - (c) Find the coordinates of the point B . *(3 marks)*

3 The graph of $y = f(x)$ is sketched below for $0 \leq x \leq 5$.



- (a) Given that f is an even function, sketch the graph of $y = f(x)$ for $-5 \leq x \leq 5$. (2 marks)
- (b) Sketch the graph of $y = 2 - f(x)$ for $0 \leq x \leq 5$. (2 marks)
- (c) Describe fully a geometrical transformation that would map the graph of $y = f(x)$ onto the graph of $y = 2 - f(x)$. (2 marks)

4 The function g has domain $-1 \leq x \leq 2$ and is defined by $g(x) = x^2 + 5$.

- (a) Find $g(-1)$ and $g(2)$. (2 marks)
- (b) Sketch the graph of $y = g(x)$. (2 marks)
- (c) Find the range of g . (3 marks)
- (d) State, with a reason, whether the inverse function, g^{-1} , exists. (2 marks)
- (e) Find $gg(x)$, giving your answer in the form $x^4 + px^2 + q$. (2 marks)

5 The quadratic equation

$$x^2 + (3 - k)x + 5 - k^2 = 0$$

is to be considered for different values of the constant k .

- (a) When $k = 7$:
- (i) show that $x^2 - 4x - 44 = 0$; (1 mark)
- (ii) find the roots of this equation, giving your answers in the form $a + b\sqrt{3}$, where a and b are integers. (2 marks)
- (b) When the quadratic equation $x^2 + (3 - k)x + 5 - k^2 = 0$ has equal roots:
- (i) show that $5k^2 - 6k - 11 = 0$; (3 marks)
- (ii) hence find the possible values of k . (2 marks)

Turn over ►

- 6 (a) Express the equation

$$5 \sin 2x - 4 \cos 2x = 0$$

in the form

$$\tan 2x = k,$$

where k is a constant.

(2 marks)

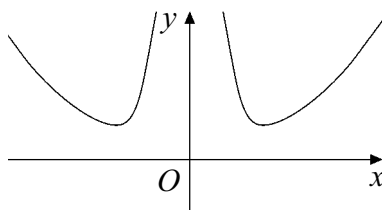
- (b) Hence find all solutions of the equation

$$5 \sin 2x - 4 \cos 2x = 0$$

in the interval $0^\circ < x < 180^\circ$, giving your answers to the nearest 0.1° .

(4 marks)

- 7 A curve has equation $y = x^2 + \frac{81}{x^2}$. Its graph is sketched below.



- (a) (i) Find $\frac{dy}{dx}$. (3 marks)
- (ii) Show that the stationary points of the curve occur when $x^4 = 81$. (2 marks)
- (iii) Hence find the x -coordinates of the stationary points. (2 marks)
- (iv) Find the value of the y -coordinate at each stationary point. (1 mark)
- (b) (i) Find $\int \left(x^2 + \frac{81}{x^2} \right) dx$. (3 marks)
- (ii) Hence find the area of the region bounded by the curve, the lines $x = 1$, $x = 3$ and the x -axis. (2 marks)

END OF QUESTIONS