

General Certificate of Education
January 2004
Advanced Level Examination



**MATHEMATICS AND STATISTICS
(SPECIFICATION B)
Unit Mechanics 5**

MBM5

Tuesday 27 January 2004 Afternoon Session

In addition to this paper you will require:

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM5.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 At time t , a force, $6e^{-2t}$, acts upon a particle which is initially at rest.

Find the momentum of the particle when $t = 4$. (4 marks)

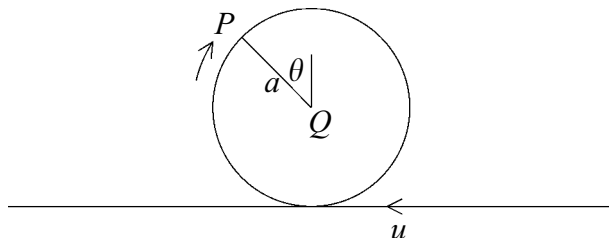
- 2 A force \mathbf{F} , of magnitude 21 N, acts in the direction AB . The coordinates of the points A and B are $(4, -3, 5)$ and $(2, -1, 4)$ respectively. The three unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are in the directions of the x , y and z axes. The unit of distance is metres.

(a) Find the force \mathbf{F} . (4 marks)

(b) Find the work done by the force \mathbf{F} when it acts on a body which moves from point P $(-1, 5, 8)$ to point Q $(4, 6, -2)$. (4 marks)

- 3 In crazy golf, a golf ball is fired along a smooth track and loops the loop inside a section of track.

Model this loop as a vertical circle of radius a and centre Q , as shown in the diagram.



The golf ball is travelling at speed u as it enters the circle at the lowest point.

Model the ball as a particle P , of mass m .

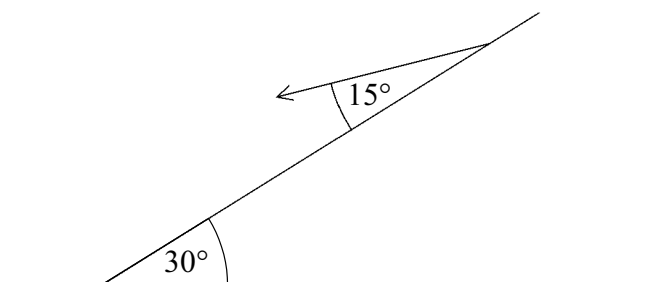
- (a) Show that the reaction of the track on the particle when QP makes an angle of θ with the upward vertical is

$$\frac{mu^2}{a} - 3mg \cos \theta - 2mg \quad (6 \text{ marks})$$

- (b) Given that the ball completes a vertical circle inside the track, show that

$$u \geq \sqrt{(5ag)} \quad (2 \text{ marks})$$

- 4 A particle is projected down a plane inclined at an angle 30° to the horizontal. It is projected with velocity V at an angle 15° to the inclined plane.



The particle moves in a vertical plane containing the line of greatest slope.

Show that the range down the plane is

$$\frac{2V^2}{3g} \quad (10 \text{ marks})$$

- 5 A rocket of initial mass 10 000 kg is launched from a space station where gravity can be ignored. At time t seconds after the launch, the mass of the rocket is m kg and it is travelling at v m s^{-1} . The burnt fuel is ejected at 600 m s^{-1} relative to the rocket and at a constant rate of 200 kg s^{-1} .

- (a) Write down the mass of the rocket at time t whilst the fuel is still burning. *(1 mark)*
- (b) By considering linear momentum, show that

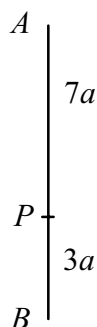
$$\frac{dv}{dt} = \frac{600}{50 - t} \quad (7 \text{ marks})$$

- (c) Given that the initial mass of fuel is 7000 kg, find the maximum acceleration of the rocket. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ►

- 6 A spring, of natural length $4a$ and modulus λ , has one end attached to a fixed support A , and a particle P of mass m is attached to its other end. Another spring, of natural length $2a$ and modulus $4mg$, has one end attached to P and the other end attached to a fixed support B , which is situated at a distance of $10a$ vertically below A . The system is in equilibrium in a vertical line with the upper spring stretched to a length of $7a$ and the lower spring stretched to a length of $3a$ as shown in the diagram.



- (a) Show that $\lambda = 4mg$. (4 marks)

- (b) At time $t = 0$, the particle is lowered to a distance $\frac{a}{2}$ below its equilibrium position and released from rest. The subsequent motion of P is subject to a resistance of magnitude $\frac{1}{5}mkv$, where $k^2 = \frac{6g}{a}$ and v is the speed of the particle at time t .

- (i) Given that x is the downward displacement of P from its equilibrium position at time t , show that

$$10 \frac{d^2x}{dt^2} + 2k \frac{dx}{dt} + 5k^2 x = 0 \quad (6 \text{ marks})$$

- (ii) Hence find x in terms of a , k and t . (9 marks)

END OF QUESTIONS