General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 4

MBM4

Wednesday 21 January 2004 Afternoon Session

In addition to this paper you will require:
- an 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 15 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM4.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A squash ball, of mass 40 grams, is travelling horizontally at $12\,\mathrm{m\,s^{-1}}$. It strikes a wall which is perpendicular to its velocity and rebounds directly with speed $8\,\mathrm{m\,s^{-1}}$.

(a) Calculate the impulse acting on the ball. *(4 marks)*

(b) The ball is in contact with the wall for 0.05 seconds. Find the magnitude of the average force acting on the ball during the time that it is in contact with the wall. *(2 marks)*

2 The acceleration, a, of a body falling with speed v and subject to air resistance may be modelled by the equation

$$a = g - \lambda v^2$$

where λ is constant.

Find the dimensions of λ in order that the equation is dimensionally consistent. *(4 marks)*

3 Three forces act at points in the x, y plane as listed below:

$7\mathbf{i}+2\mathbf{j}$	at	$(3, -5)$
$-3\mathbf{i}+4\mathbf{j}$	at	$(4, 1)$
$\mathbf{i}+6\mathbf{j}$	at	$(8, 2)$

The three forces are equivalent to a single force, $\mathbf{F}$.

(a) Find the magnitude of $\mathbf{F}$. *(4 marks)*

(b) Show that the line of action of $\mathbf{F}$ acts through the point $(\frac{53}{6}, 0)$. *(5 marks)*

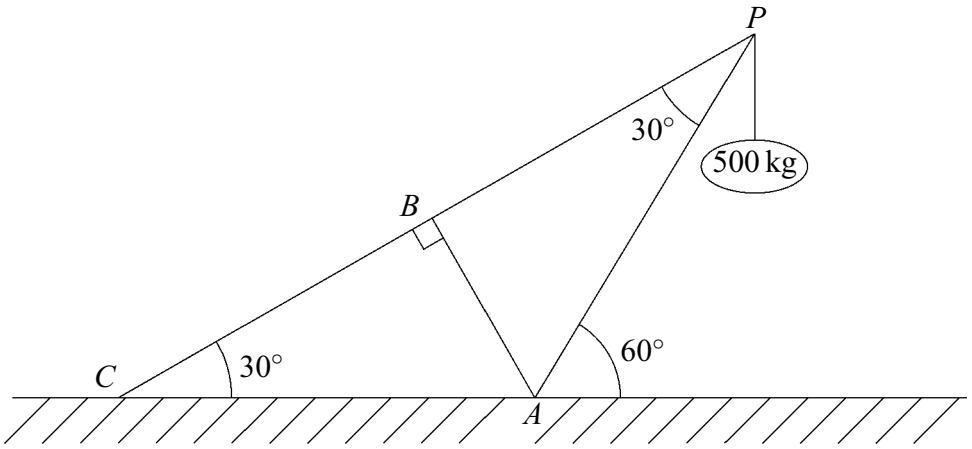
4 A smooth sphere, A, has mass $3m$ and velocity $7\mathbf{i} - 8\mathbf{j}$. It collides with a second smooth sphere, B, which has mass m and velocity $2\mathbf{i}+5\mathbf{j}$. The two spheres have the same radius. After the collision, the velocity of B is $5\mathbf{i} - 4\mathbf{j}$.

(a) Find the velocity of A after the collision. *(4 marks)*

(b) Find the change in momentum of B. *(2 marks)*

(c) Find, as a vector, the direction of the line of centres of the spheres during the collision. Give a reason for your answer. *(2 marks)*

5 A framework is composed of four light, inextensible, smoothly jointed rods, *AP*, *BP*, *CB* and *AB*. The framework is fixed at *A* and *C*, and *CBP* lies in a straight line. The framework stands in a vertical plane and a load of mass 500 kg is hung from *P*. The framework and load are shown in the diagram below.

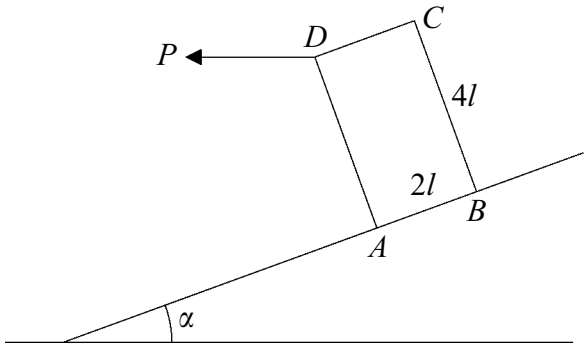


(a) Find the forces in each of the rods *AP* and *BP*. State whether the rods are in tension or compression. *(6 marks)*

(b) State the force in the rod *AB*. Give a reason for your answer. *(2 marks)*

6 A uniform solid cuboid, of mass *m*, is placed on a rough inclined plane and remains at rest. The cuboid has a square base of side $2l$ and a height of $4l$. A **horizontal** force *P*, which is gradually increasing, is then applied to the mid-point of, and perpendicular to, a top edge.

This force acts as shown in the diagram, where *ABCD* is a vertical cross-section through the centre of mass of the cuboid.



The angle which the inclined plane makes with the horizontal is α. The coefficient of friction between the block and the plane is μ.

(a) Find *P*, if the block is on the point of toppling. *(4 marks)*

(b) Find *P*, if the block is on the point of sliding. *(7 marks)*

7 The unit vectors **i** and **j** are defined in the east and north directions respectively. The unit of distance is kilometres and the unit of velocity is kilometres per hour.

Initially, two ships P and Q are 2 kilometres apart with P due south of Q.

Ship Q is travelling with velocity $10\sqrt{3}\mathbf{i} - 10\mathbf{j}$ kilometres per hour.

The maximum speed of ship P is 8 kilometres per hour.

(a) Find the speed of ship Q, and the bearing on which it is travelling. *(3 marks)*

(b) Ship P travels to ensure that it approaches Q as closely as possible.

(i) Find the direction in which P travels. *(4 marks)*

(ii) Show that the velocity of Q relative to P is $11\mathbf{i} - 15\mathbf{j}$ correct to 2 significant figures. *(3 marks)*

(iii) Find the shortest distance between the ships. *(4 marks)*

END OF QUESTIONS