General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B)
Unit Mechanics 3

MBM3

Friday 23 January 2004 Morning Session

In addition to this paper you will require:
- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM3.
- Answer **all** questions.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$ unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 As a lift moves upwards its motion has three stages.

Stage I It accelerates uniformly, from rest, at $0.2\,\text{m s}^{-2}$, for 8 seconds.
Stage II It then travels at a constant speed for 3 seconds.
Stage III It then decelerates uniformly, stopping when it has travelled a further 1 metre.

(a) (i) Find the distance travelled by the lift in stage I. *(2 marks)*

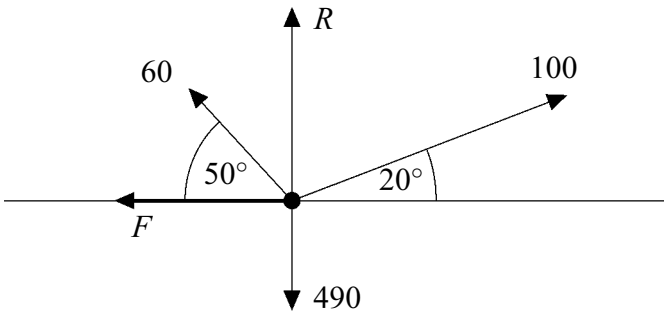
(ii) Find the speed of the lift at the end of stage I. *(2 marks)*

(iii) Find the total distance travelled by the lift. *(2 marks)*

(b) Find the total time for which the lift is moving. *(3 marks)*

(c) The lift and its passengers have a total mass of 600 kg. The lift is supported by a single cable. Find the tension in the cable when the lift is accelerating upwards at $0.2\,\text{m s}^{-2}$. *(3 marks)*

2 A box, of mass 50 kg, moves in a straight line on a rough horizontal surface. The diagram below shows **all** the forces acting on the box, as it moves. The magnitude of each force is in newtons. The box is modelled as a particle.



Note that the weight of the box has been included in the diagram.

(a) Find R, the magnitude of the normal reaction force acting on the box. *(3 marks)*

(b) The box accelerates at $0.5\,\text{m s}^{-2}$.

(i) Show that F, the magnitude of the friction force acting on the box, is approximately 30.4 . *(4 marks)*

(ii) Find the coefficient of friction between the box and the surface. *(2 marks)*

3 A diver has mass 65 kg. She dives from a fixed diving board, which is 6 metres above the level of the water in the pool. When the diver leaves the board, she is travelling vertically upwards and has speed $2\,\mathrm{m\,s^{-1}}$.

Model the diver as a particle. Assume that there are no resistance forces acting on the diver as she moves through the air and that she does not hit the board on the way down.

(a) (i) Calculate the kinetic energy of the diver when she leaves the board. *(2 marks)*

(ii) By using an energy method, calculate the maximum height of the diver above the diving board. *(2 marks)*

(b) (i) Find the kinetic energy of the diver when she hits the water. *(3 marks)*

(ii) Hence calculate the speed of the diver when she hits the water. *(2 marks)*

4 A particle moves on a straight line. At time t seconds its acceleration, $a\,\mathrm{m\,s^{-2}}$, is given by

$$a = 20 \sin 4t$$

(a) Initially the particle is at rest. Find an expression for the velocity of the particle at time t. *(4 marks)*

(b) Initially the displacement of the particle from the origin is 0.8 metres. Find an expression for the displacement of the particle at time t. *(4 marks)*

5 A particle moves in a horizontal plane. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular and lie in the horizontal plane.

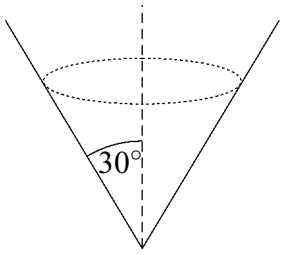
Initially the particle is at the origin and has velocity $(5\mathbf{i} - 2\mathbf{j})\ \mathrm{m\,s^{-1}}$. It moves with constant acceleration. At time $t = 10$ it has position vector $(40\mathbf{i} - 15\mathbf{j})$ metres.

(a) Show that the velocity of the particle is $(3\mathbf{i} - \mathbf{j})\ \mathrm{m\,s^{-1}}$ when $t = 10$. *(4 marks)*

(b) Find the acceleration of the particle. *(3 marks)*

(c) The mass of the particle is 15 kg. Find the magnitude of the resultant force acting on the particle. *(4 marks)*

6 A particle, of mass 3 kg, describes a horizontal circular path on the inside surface of a smooth cone, as shown in the diagram.



The radius of the circle is 0.5 metres and the semi-vertical angle of the cone is 30°. The particle moves at a constant speed.

(a) (i) Show that the magnitude of the normal reaction force on the particle is 58.8 N. *(3 marks)*

(ii) Find the speed of the particle. *(4 marks)*

(b) The particle moves on the same cone, but in a horizontal circle of greater radius than before.

(i) What happens to the magnitude of the normal reaction force? *(1 mark)*

(ii) What happens to the speed of the particle? Explain your answer. *(2 marks)*

7 An elastic string has natural length 2 metres and modulus of elasticity λ newtons. One end of the string is fixed at the point O, and a particle of mass 20 kg is attached to the other end of the string.

(a) When in equilibrium the particle is 2.7 metres below O. Show that $\lambda = 560$. *(3 marks)*

(b) The particle is now held at O and released from rest. The maximum length of the string in the subsequent motion is L.

(i) Show that L satisfies the equation

$$5L^2 - 27L + 20 = 0$$ *(5 marks)*

(ii) Find the maximum length of the string. *(3 marks)*

8 A cyclist moves from rest along a straight horizontal road. At time t seconds, the displacement of the cyclist from his initial position is s metres.

(a) For $0 \leqslant t \leqslant 10$,

$$s = \frac{t^4}{400} - \frac{t^3}{10} + \frac{3t^2}{2}$$

(i) Find s when $t = 10$. *(1 mark)*

(ii) Find the velocity of the cyclist when $t = 10$. *(3 marks)*

(iii) Find the acceleration of the cyclist when $t = 10$. *(3 marks)*

(b) For $t \geqslant 10$ the cyclist moves with a constant velocity, so that

$$s = ht - k$$

where h and k are constants. Find the values of h and k. *(3 marks)*

END OF QUESTIONS