

General Certificate of Education  
January 2004  
Advanced Level Examination



**MATHEMATICS AND STATISTICS  
(SPECIFICATION B)  
Unit Mechanics 2**

**MBM2**

Friday 23 January 2004 Morning Session

**In addition to this paper you will require:**

- a 12-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBM2.
- Answer **all** questions.
- Take  $g = 9.8 \text{ m s}^{-2}$  unless stated otherwise.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

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Answer **all** questions.

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- 1 A diver has mass 65 kg. She dives from a fixed diving board, which is 6 metres above the level of the water in the pool. When the diver leaves the board, she is travelling vertically upwards and has speed  $2 \text{ m s}^{-1}$ .

Model the diver as a particle. Assume that there are no resistance forces acting on the diver as she moves through the air and that she does not hit the board on the way down.

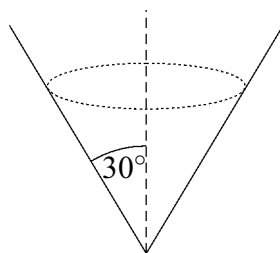
- (a) (i) Calculate the kinetic energy of the diver when she leaves the board. (2 marks)
- (ii) By using an energy method, calculate the maximum height of the diver above the diving board. (2 marks)
- (b) (i) Find the kinetic energy of the diver when she hits the water. (3 marks)
- (ii) Hence calculate the speed of the diver when she hits the water. (2 marks)

- 2 A particle moves on a straight line. At time  $t$  seconds its acceleration,  $a \text{ m s}^{-2}$ , is given by

$$a = 20 \sin 4t$$

- (a) Initially the particle is at rest. Find an expression for the velocity of the particle at time  $t$ . (4 marks)
- (b) Initially the displacement of the particle from the origin is 0.8 metres. Find an expression for the displacement of the particle at time  $t$ . (4 marks)

- 3 A particle, of mass 3 kg, describes a horizontal circular path on the inside surface of a smooth cone, as shown in the diagram.



The radius of the circle is 0.5 metres and the semi-vertical angle of the cone is  $30^\circ$ . The particle moves at a constant speed.

- (a) (i) Show that the magnitude of the normal reaction force on the particle is 58.8 N. *(3 marks)*
- (ii) Find the speed of the particle. *(4 marks)*
- (b) The particle moves on the same cone, but in a horizontal circle of greater radius than before.
- (i) What happens to the magnitude of the normal reaction force? *(1 mark)*
- (ii) What happens to the speed of the particle? Explain your answer. *(2 marks)*
- 4 An elastic string has natural length 2 metres and modulus of elasticity  $\lambda$  newtons. One end of the string is fixed at the point  $O$ , and a particle of mass 20 kg is attached to the other end of the string.

- (a) When in equilibrium the particle is 2.7 metres below  $O$ . Show that  $\lambda = 560$ . *(3 marks)*
- (b) The particle is now held at  $O$  and released from rest. The maximum length of the string in the subsequent motion is  $L$ .
- (i) Show that  $L$  satisfies the equation

$$5L^2 - 27L + 20 = 0 \quad \text{span style="float: right;">*(5 marks)*$$

- (ii) Find the maximum length of the string. *(3 marks)*

5 A cyclist moves from rest along a straight horizontal road. At time  $t$  seconds, the displacement of the cyclist from his initial position is  $s$  metres.

(a) For  $0 \leq t \leq 10$ ,

$$s = \frac{t^4}{400} - \frac{t^3}{10} + \frac{3t^2}{2}$$

(i) Find  $s$  when  $t = 10$ . *(1 mark)*

(ii) Find the velocity of the cyclist when  $t = 10$ . *(3 marks)*

(iii) Find the acceleration of the cyclist when  $t = 10$ . *(3 marks)*

(b) For  $t \geq 10$  the cyclist moves with a constant velocity, so that

$$s = ht - k$$

where  $h$  and  $k$  are constants. Find the values of  $h$  and  $k$ . *(3 marks)*

6 A uniform lamina is bounded by the  $x$ -axis, and the lines  $x = a$  and  $y = kx$ , where  $k$  is a positive constant.

(a) Find the area of the lamina in terms of  $a$  and  $k$ . *(2 marks)*

(b) Use integration to show that the  $x$ -coordinate of the centre of mass of the lamina is  $\frac{2a}{3}$ . *(4 marks)*

(c) Use integration to find the  $y$ -coordinate of the centre of mass of the lamina. *(4 marks)*

7 A particle moves with simple harmonic motion on a straight line between two points  $A$  and  $B$ , which are 0.4 metres apart. The maximum speed of the particle is  $10 \text{ m s}^{-1}$ .

(a) Show that the period of the motion is  $\frac{\pi}{25}$  seconds. (4 marks)

(b) Find the speed of the particle when it is 0.04 metres from  $A$ . (3 marks)

(c) The distance,  $s$ , of the particle from  $A$  at time  $t$  is given by

$$s = p - q \cos(\omega t)$$

where  $\omega$ ,  $p$  and  $q$  are constants.

(i) State the values of  $\omega$  and  $q$ . (2 marks)

(ii) When  $t = 0$  the particle is at  $A$ . Find the value of  $p$ . (2 marks)

8 A stone, of mass 0.1 kg, is projected vertically upwards from a catapult at a speed of  $12 \text{ m s}^{-1}$ . As the stone rises it is acted on by gravity and air resistance. When the stone is moving at  $v \text{ m s}^{-1}$ , the air resistance has magnitude  $\frac{v^2}{200}$  newtons.

(a) The height of the stone above the level of projection at time  $t$  seconds is  $x$  metres. Show that while the stone is rising

(i)  $v \frac{dv}{dx} = -\left(9.8 + \frac{v^2}{20}\right)$  (2 marks)

(ii)  $10 \ln\left(9.8 + \frac{v^2}{20}\right) + x = 10 \ln 17$  (6 marks)

(b) Find the maximum height of the stone. (3 marks)

**END OF QUESTIONS**