General Certificate of Education
January 2004
Advanced Subsidiary Examination

MATHEMATICS AND STATISTICS (SPECIFICATION B) **MBD1**
Unit Discrete 1

Monday 12 January 2004 Afternoon Session

In addition to this paper you will require: • a 12-page answer book;   • the AQA booklet of formulae and statistical tables;   • an insert for use in Questions 1, 2, 3, 5 and 6 (enclosed);   • a ruler.   You may use a graphics calculator.

Time allowed: 1 hour 45 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MBD1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Fill in the boxes at the top of the insert. Make sure that you attach the insert to your answer book.

Information

- The maximum mark for this paper is 80.
- Mark allocations are shown in brackets.

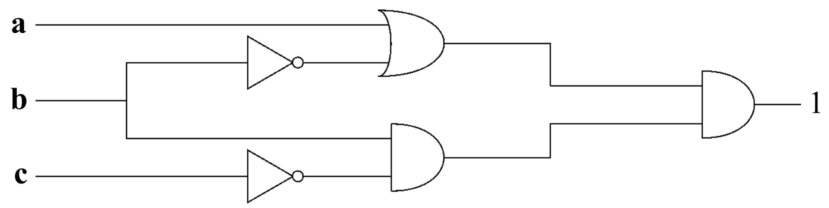
Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

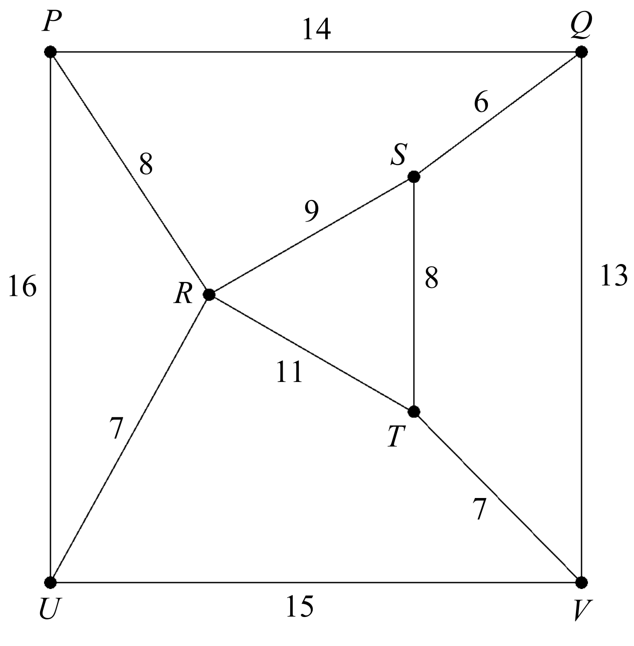
In the following combinatorial circuit the output is 1.



By tracing back through the circuit on **Figure 1**, or otherwise, find the values of **a**, **b** and **c**. *(5 marks)*

2 [**Figure 2**, printed on the insert, is provided for use in answering part (a) of this question.]

The network below represents seven campsites $P - V$ and the footpaths between them. The numbers on the arcs are the lengths of those footpaths in miles.



(a) Use Dijkstra's algorithm on **Figure 2** to find the shortest route from P to V on footpaths. Show your working clearly at each vertex. *(6 marks)*

(b) Some walkers on a camping holiday spend each night at a campsite and can only walk up to 10 miles a day. What is the minimum number of days which they need to get from P to V? Give a reason for your answer. *(2 marks)*

3 [**Figure 3**, printed on the insert, is provided for use in answering part (a) of this question.]

The walking times, in minutes, between six points *A*–*F* are given in the following table:

	A	***B***	***C***	***D***	***E***	***F***
A	–	20	30	20	15	10
B	20	–	35	30	25	25
C	30	35	–	30	30	25
D	20	30	30	–	15	25
E	15	25	30	15	–	10
F	10	25	25	25	10	–

(a) Use Prim's algorithm on **Figure 3**, starting at *A*, to find a minimum connector of the six points. *(5 marks)*

(b) Draw a tree representing the minimum connector which you found in part (a). *(2 marks)*

(c) Due to poor weather, only the footpaths in the minimum connector are open. Use your tree from part (b) to show that it is still possible to walk from any one of the six points to any other in less than 1 hour. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

4 Let **p**, **q** and **r** denote the folowing statements:

p: a student is married;
q: a student is under 18;
r: a student is eligible for a grant.

(a) Express $\mathbf{r} \Rightarrow (\mathbf{p} \wedge \mathbf{q})$ as a sentence using the above statements. *(2 marks)*

(b) Express the following sentence in terms of **p**, **q**, **r**, $\vee$, $\Rightarrow$ and $\sim$:

"If a student is not married or is not under 18 then the student is not eligible for a grant." *(2 marks)*

(c) (i) Copy and complete the following truth table, with as many extra columns as you wish, to show the values of the expressions in parts (a) and (b).

p	q	r	$\mathbf{p} \wedge \mathbf{q}$	(a) $\mathbf{r} \Rightarrow (\mathbf{p} \wedge \mathbf{q})$	....	(b)
0	0	0	0	1		
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

(6 marks)

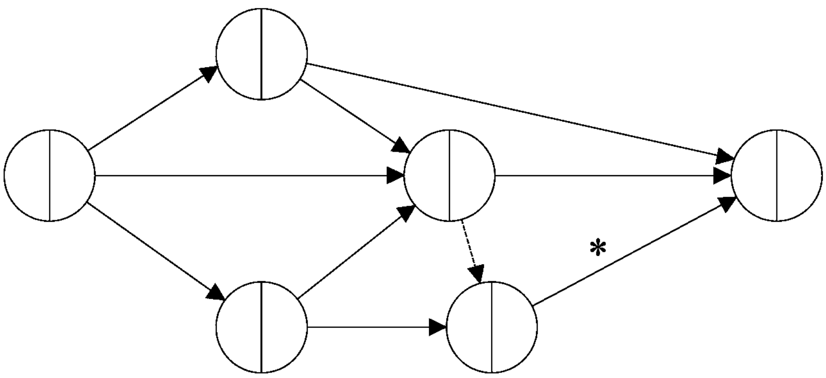
(ii) Hence state whether the two expressions in parts (a) and (b) are equivalent. *(1 mark)*

5 [**Figure 4**, printed on the insert, is provided for use in answering this question.]

A project is made up of activities A – I. Their duration, in hours, and their immediate predecessors are given in the table.

Activity	**Duration** (hours)	**Cannot start until these activities are completed**
A	10	–
B	5	–
C	6	–
D	3	C
E	6	C
F	4	B
G	11	B
H	8	A, D, F
I	10	A, D, E, F

An activity network for this project is given below, but the labels have been omitted from the arcs.



(a) By considering the number of its immediate predecessors, show that the arc marked * must represent activity I. Hence label all the arcs in **Figure 4**. *(4 marks)*

(b) Perform a forward and backward pass on your activity network in order to calculate all the early and late event times. *(5 marks)*

(c) State the minimum completion time for the project and list the critical activities. *(2 marks)*

(d) Calculate the independent float of activity H. *(2 marks)*

(e) Extra workers are employed to cut the duration of activity E by 50%. By how much does this reduce the minimum completion time of the project? *(3 marks)*

Turn over ▶

6 [**Figure 5**, which consists of graph paper with the axes drawn, is printed on the insert for use in answering part (b) of this question.]

At Basehome Do-it-yourself Store, nails, screws and hooks can be bought in two types of mixed pack, the Xperience pack and the You-do-it pack.

The Xperience pack contains 1000 nails, 1500 screws and 500 hooks. It costs £6.
The You-do-it pack contains 100 nails, 100 screws and 100 hooks. It costs £1.
A housebuilder needs 5000 nails, 6000 screws and 3000 hooks.

Assume that the housebuilder buys x Xperience packs and y You-do-it packs.

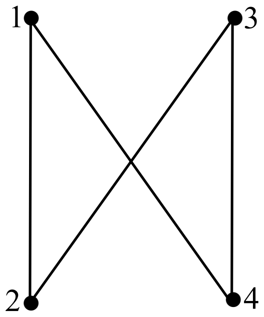
(a) Show that the inequality $10x + y \geqslant 50$ ensures that the housebuilder has enough nails and write down the corresponding inequalities for the screws and the hooks. *(4 marks)*

(b) On **Figure 5** illustrate the region of those x and y which satisfy $x \geqslant 0$, $y \geqslant 0$ and the three inequalities from part (a). *(5 marks)*

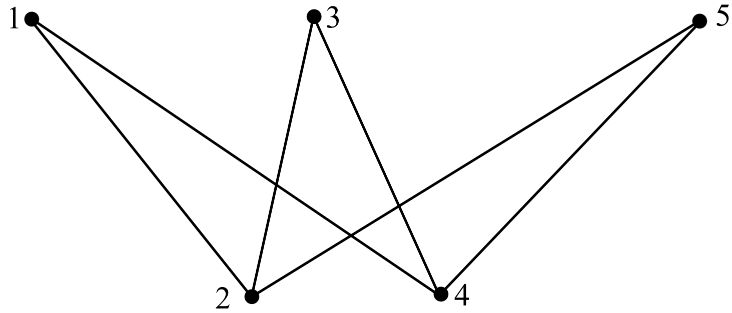
(c) Calculate how many packs of each type the housebuilder should buy in order to satisfy his requirements at the minimum cost. *(4 marks)*

(d) The Basehome Do-it-yourself Store has a "3-for-the-price-of-2" offer: if you buy 3 identical items you only have to pay for 2 of them. If the housebuilder takes advantage of this offer, how many packs of each type should he buy in order to satisfy his requirements at minimum cost? *(3 marks)*

7 $\mathbf{G}_n$ is the graph with n vertices labelled 1, 2, 3, …, n and with two vertices joined by an edge if the sum of their labels is odd. So, for example, $\mathbf{G}_4$ and $\mathbf{G}_5$ are as drawn below.



$\mathbf{G}_4$



$\mathbf{G}_5$

(a) Draw $\mathbf{G}_6$. *(2 marks)*

(b) (i) Give an example of a Hamiltonian cycle in $\mathbf{G}_6$. *(2 marks)*

(ii) For what values of n is $\mathbf{G}_n$ Hamiltonian? *(2 marks)*

(c) (i) Explain how you know that $\mathbf{G}_6$ is not planar. *(1 mark)*

(ii) For what values of n is $\mathbf{G}_n$ planar? Give reasons for your answer. *(3 marks)*

(d) If n is even what, in terms of n, is the degree of each vertex of $\mathbf{G}_n$? *(1 mark)*

(e) For what values of n is $\mathbf{G}_n$ Eulerian? Justify your answer. *(3 marks)*

END OF QUESTIONS