General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAS3**
Unit Statistics 3

Friday 24 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The weights of oranges sold by a greengrocer are known to be normally distributed. The greengrocer takes a random sample of 10 oranges and records their weights in grams. The results are as follows.

176 194 193 191 170 177 178 160 188 169

(a) (i) Calculate unbiased estimates of the mean and variance of the weights of oranges, giving your answers to one decimal place. *(3 marks)*

(ii) Construct a 90% confidence interval for the mean weight of oranges. *(5 marks)*

(b) The greengrocer currently sells the oranges by weight at a price of £1.20 **per kilogram**.

(i) Deduce a 90% confidence interval for the mean price per orange. *(2 marks)*

(ii) The greengrocer decides to sell the oranges individually at a price of 20 pence per orange. Use the confidence interval you found in part (b)(i) to comment on this decision. *(2 marks)*

2 Ann and Brian usually travel to work on the same train. The train is scheduled to leave the platform at 8.00 am, but it actually leaves at time T minutes after 8.00 am. The random variable T has an exponential distribution with distribution function:

$$\mathrm{F}(t) = \begin{cases} 1 - \mathrm{e}^{-\frac{t}{4}} & t \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Write down the probability that the train leaves the platform at exactly 8.00 am. *(1 mark)*

(b) One morning Ann arrives on the platform at 8.02 am and Brian arrives at 8.06 am.

(i) Find the probability that the train has left before Ann arrives. *(2 marks)*

(ii) Find the probability that Ann catches the train but Brian misses it. *(3 marks)*

(c) Another morning Brian arrives on the platform at 8.01 am and is in time to catch the train. Ann arrives at 8.04 am. Find the probability that Ann also catches the train. *(4 marks)*

3 Daksha is investigating memory retention by students. She gives a list of 30 words to each of a random sample of 10 students and asks each student to memorise as many words as possible in a fifteen-minute period.

Daksha then asks each student to write down as many words as possible from the list. Three days later, she again asks each student to write down as many words as possible.

Daksha then calculates, for each student, the decrease, d, in the number of words recalled after a lapse of three days. Her results are as follows.

Student	A	B	C	D	E	F	G	H	I	J
d	6	4	4	3	1	5	2	0	8	7

Daksha predicts that the median value of d for all students is 5. Use a Wilcoxon signed-rank test and the 10% level of significance to investigate her prediction. *(9 marks)*

4 Evan is a self-employed decorator. Before he starts each job, he visits the site and estimates the time he will need to complete the work. At the end of each job, he records the actual time taken and calculates the percentage error, X, in his estimate.

Evan finds that the mean value of X is approximately zero, but that X has a high standard deviation.

(a) Based on a random sample of 9 jobs, an unbiased estimate of the population variance, ${\sigma_X}^2$, of X is ${s_X}^2 = 470.3$.

Assuming that X is normally distributed, investigate, at the 5% level of significance, Evan's belief that $\sigma_X > 15$. *(6 marks)*

(b) Evan decides that he can reduce the standard deviation of his percentage errors by inspecting each site more thoroughly before he makes his estimate.

Under this new system, he calculates the percentage error, Y, in his estimates. Based on a random sample of 7 jobs, an unbiased estimate of the population variance, ${\sigma_Y}^2$, of Y is ${s_Y}^2 = 136.3$.

Assuming that Y is also normally distributed, use an appropriate test, at the 5% level of significance, to determine whether Evan has reduced the standard deviation of his percentage errors. *(6 marks)*

5 The speeds, X mph, of cars travelling along a road subject to a 30 mph speed limit are recorded.

In an attempt to reduce the speed of cars on this road, a display screen is installed to show the speed of each car as it approaches the screen. The speeds, Y mph, of cars travelling along the road are then recorded.

You may assume that X and Y are normally distributed random variables, each with a standard deviation of 2.7.

(a) For a random sample of 10 cars before the display screen was installed, the mean speed was 37.6 mph. For a random sample of 10 cars after the display screen was installed, the mean speed was 31.3 mph.

(i) Construct a 98% confidence interval for the mean decrease in speed of cars after the display screen was installed. *(5 marks)*

(ii) Comment on the claim that the display screen is effective in reducing the mean speed of cars. *(1 mark)*

(iii) Find the width of your confidence interval calculated in part (a)(i). *(1 mark)*

(b) A local councillor has to report on the effects of installing the screen. She wants to quote a confidence interval of smaller width for the mean decrease in speed of cars. The councillor considers two methods of obtaining this confidence interval.

Method 1: Use the original samples of cars, each of size 10, and reduce the level of confidence to α %.

Method 2: Take two larger random samples of cars, each of size n, with one sample taken before and one taken after the screen was installed. Calculate a 98% confidence interval for the mean decrease in speed.

(i) Show that, with α equal to 78, the width of the confidence interval found using **Method 1** is less than 3 mph. *(3 marks)*

(ii) Find the smallest possible value of n so that the width of the confidence interval found using **Method 2** is less than 3 mph. *(5 marks)*

(iii) State, with a reason, which of the two methods would give the more reliable estimate for the mean decrease in speed of cars. *(2 marks)*

END OF QUESTIONS