General Certificate of Education
June 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP5**
Unit Pure 5

Thursday 9 June 2005 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Find the integrating factor for the differential equation

$$\frac{dy}{dx} + y = e^x.$$ *(2 marks)*

(b) Find the general solution of this differential equation, giving your answer in the form $y = f(x)$. *(4 marks)*

2 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $$f(x, y) = \frac{\sin(x + y)}{xy}$$

and $$y(1) = 0.8\,.$$

Obtain estimates, to **four** decimal places, of the value of $y(1.05)$:

(a) using the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.05$; *(3 marks)*

(b) using the formula

$$y_{r+1} = y_r + \tfrac{1}{2}(k_1 + k_2)$$

where $$k_1 = hf(x_r, y_r),$$

$$k_2 = hf(x_r + h, y_r + k_1)$$

and $h = 0.05$. *(7 marks)*

3 Find

$$\lim_{x \to 0} \left(\frac{x \ln(1+x)}{1 - \cos x} \right).$$

(4 marks)

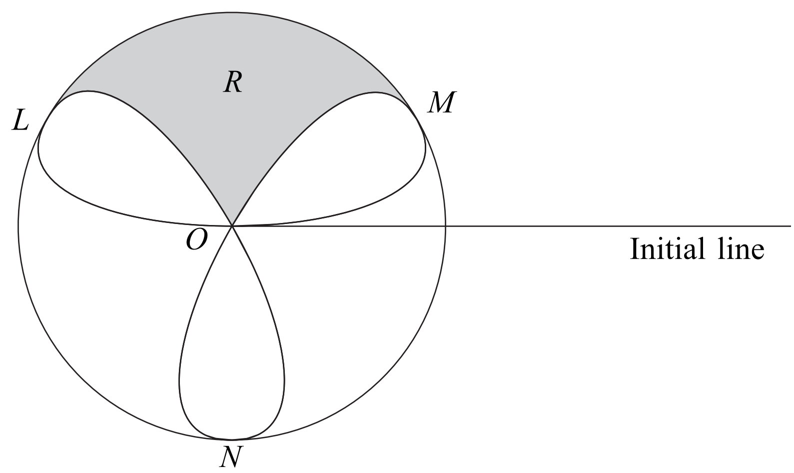
4 The diagram shows the curve C_1 with polar equation

$$r = \sin 3\theta$$

and the circle C_2 with polar equation

$$r = 1$$

which touches C_1 at the points L, M and N.



(a) For the curve C_1:

(i) find the values of θ, $0 \leqslant \theta < \frac{1}{2}\pi$, for which $r = 0$; *(2 marks)*

(ii) show that the area of one of its loops is $\frac{1}{12}\pi$. *(4 marks)*

(b) Find the area of the shaded region R, bounded by two arcs OL and OM of the curve C_1 and the arc LM of the circle C_2. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

5 (a) Using the substitution $u = \ln x$, or otherwise, find

$$\int \frac{1}{x \ln x} \, dx .$$

(3 marks)

(b) (i) Explain why

$$\int_1^e \frac{1}{x \ln x} \, dx$$

is an improper integral. *(1 mark)*

(ii) Determine whether

$$\int_1^e \frac{1}{x \ln x} \, dx$$

exists, giving a reason for your answer. *(2 marks)*

6 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 3 \cos x + 4 \sin x .$$

(11 marks)

7 (a) (i) Show that the equation

$$3x^2 + 4x + 4y^2 = 4$$

is equivalent to

$$(2 - x)^2 = 4(x^2 + y^2)\,.$$ *(2 marks)*

(ii) Hence show that the polar equation of the curve

$$3x^2 + 4x + 4y^2 = 4$$

is $$\frac{2}{r} = 2 + \cos\theta\,.$$ *(6 marks)*

(b) A straight line through the pole O meets the curve

$$\frac{2}{r} = 2 + \cos\theta$$

at the points P and Q.

Show that

$$\frac{1}{OP} + \frac{1}{OQ} = 2\,.$$ *(6 marks)*

END OF QUESTIONS

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