Version 1.0: 0805

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6300
Specification A

MAP5 Pure 5

Mark Scheme

2005 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	accuracy
E	mark is for	explanation
√or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
–*x* EE		deduct *x* marks for each error
NMS		no method shown
PI		possibly implied
SCA		substantially correct approach
c		candidate
sf		significant figure(s)
dp		decimal place(s)

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae book

Application of Mark Scheme

No method shown:	
Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise
More than one method / choice of solution:	
2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP5

Q	Solution	Marks	Total	Comments
1(a)	$\text{IF} = \text{e}^{\int 1\text{d}x} = \text{e}^{x}$	M1A1	2	if $\text{IF} = \text{e}^{-x}$, mark (a) M1A0 (b) M1A1A0A1
(b)	$\frac{\text{d}}{\text{d}x}\left(y\text{e}^{x}\right) = \text{e}^{2x}$	M1A1F		√ incorrect (a) if $\frac{\text{d}}{\text{d}x}\left(y\text{e}^{x}\right) = \text{e}^{x}$, mark M1A0A0A1
	$y\text{e}^{x} = \frac{1}{2}\text{e}^{2x} + C$	A1F		condone C missing
	$y = \frac{1}{2}\text{e}^{x} + C\text{e}^{-x}$	A1F	4	
	Total		**6**	
2(a)	$y(1.05) = y(1) + \frac{0.05\sin(1.8)}{1\times 0.8}$	M1A1		If worked in degrees, mark (a) M1A1A0 (b) B0M1A1A0, M1A1A1
	$= 0.8608(65476)$			
	$= 0.8609(4\text{ dp})$	A1	3	
(b)	$k_1 = \frac{0.05\sin(1+0.8)}{1\ \times\ 0.8}$ $= 0.06086(5476)$	B1		
	$k_2 = \frac{0.05\sin(1.05+0.8+0.06087)}{1.05\times 0.86087}$	M1A1F		SC: If $k_2 = \frac{0.05\sin 1.85}{1.05\times 0.85}$, mark M1A0 and ft
	$= 0.05214(7)$	A1F		
	$y(1.05) = 0.8 + \frac{1}{2}[0.06087 + 0.05215]$	M1A1F		dependent on first two M marks
	$= 0.8565(4\text{ dp})$	A1F	7	
	Total		**10**	

MAP5 (cont)

Q	Solution	Marks	Total	Comments
3	$\ln(1+x) \simeq x - \frac{x^2}{2}$			
	and $\cos x \simeq 1 - \frac{x^2}{2}$ both used	M1		
	$\lim\limits_{x\to 0}\frac{x\ln(1+x)}{1-\cos x} \simeq \lim\limits_{x\to 0}\frac{x^2+0(x^3)}{\frac{x^2}{2}+0(x^4)}$	A1		ignore errors in powers of $x > 2$
	Dividing by x^2, ie $\lim\limits_{x\to 0}\frac{1+0(x)}{\frac{1}{2}+0(x^2)}$	m1		PI
	$= 2$	A1F	4	**Alternative method:** L'Hôpital's rule used twice M1 Correct differentiation A1 Putting $x = 0$ m1 (allow this m1 if $x = 0$ gives a finite limit) Result 2 A1F
	Total		**4**	
4(a)(i)	$\sin 3\theta = 0$ when $\theta = 0, \ \frac{1}{3}\pi$	B1B1	2	
(ii)	Area of loop $= \frac{1}{2}\int_0^{\frac{1}{3}\pi} \sin^2 3\theta \, \mathrm{d}\theta$	M1		ignore limits
	$= \frac{1}{2}\int_0^{\frac{1}{3}\pi}\frac{1}{2}(1-\cos 6\theta)\mathrm{d}\theta$	m1		
	$= \frac{1}{4}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\frac{1}{3}\pi}$	A1		
	$= \frac{1}{12}\pi$	A1	4	AG
(b)	Area of $R = \frac{1}{3}\left(\pi\times 1^2 - 3\times\frac{\pi}{12}\right)$	B1B1		B1 for each part, ie for two relevant areas to be subtracted
	$= \frac{1}{4}\pi$	B1	3	OE; must be correct
	Total		**9**	

MAP5 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$u = \ln x \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$	B1		
	$\mathrm{I} = \int \frac{\mathrm{d}u}{u} = \ln u = \ln(\ln x) + c$	M1A1	3	M1 for $\ln u$ A1 for $\ln(\ln x)$ condone omission of c
(b)(i)	Clear reason why improper	E1	1	
(ii)	When $x = 1$, $\ln(\ln 1) = \ln 0$ and does not exist	E2	2	E2 for clear reasoning, E1 if vague
	Total		**6**	
6	CF $m^2 - m - 2 = 0$	M1		
	$m = 2, -1$	A1		
	CF is $Ae^{2x} + Be^{-x}$	A1F		provided roots real
	PI try $y = a\cos x + b\sin x$	M1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -a\sin x + b\cos x$	A1		
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -a\cos x - b\sin x$	A1F		allow A1 if consistent sign errors
	Substitute into DE	M1		
	$-3a - b = 3$ and $a - 3b = 4$	A1		if two mistakes in the simultaneous equations, lose this A1 and one A1F
	$a = -\frac{1}{2},\ b = -1\frac{1}{2}$	A1FA1F		If $y = 3a\cos x + 4b\sin x$ used, allow the A1Fs only if $3a = -\frac{1}{2}$ and $4b = -1\frac{1}{2}$
	GS $y = Ae^{2x} + Be^{-x} - \frac{1}{2}\cos x - \frac{3}{2}\sin x$	B1F		
	Total		**11**	

MAP5 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$4x^2+4y^2=x^2-4x+4$	M1		or any correct method
	$4\left(x^2+y^2\right)=\left(2-x\right)^2$	A1	2	AG
(ii)	Use of $x=r\cos\theta,\ x^2+y^2=r^2$	M1M1		
	$4r^2=\left(2-r\cos\theta\right)^2$	A1		
	$2r=\pm\left(2-r\cos\theta\right)$	A1		condone omission of $\pm$ sign
	$\frac{2}{r}=2+\cos\theta$	m1A1	6	AG
(b)	If $\theta=\alpha$ at P, $\theta=\alpha+\pi$ at Q	M1A1		or $x-\pi$ at Q
	$\left.\begin{aligned}\frac{2}{OP}&=2+\cos\alpha\\ \frac{2}{OQ}&=2+\cos\left(\alpha+\pi\right)\end{aligned}\right\}$	M1		
	$=2-\cos\alpha$	A1		
	$\frac{2}{OP}+\frac{2}{OQ}=4$	M1		
	$\frac{1}{OP}+\frac{1}{OQ}=2$	A1	6	AG
	Total		**14**	
	Total		**60**	