

General Certificate of Education
June 2005
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 2

MAP2

Thursday 16 June 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The geometric sequence u_1, u_2, u_3, \dots is defined by

$$u_{n+1} = u_n + \frac{2}{3}3^n, \quad \text{where } u_1 = 1.$$

(a) Write down the values of u_2, u_3, u_4 and u_5 . (3 marks)

(b) Hence write down the n th term of this sequence in terms of n . (1 mark)

(c) Show that the sum of the first 100 terms of the sequence is approximately 2.58×10^{47} . (2 marks)

2 (a) Express $\frac{4x+1}{2x-1}$ in the form $A + \frac{B}{2x-1}$, where A and B are constants. (3 marks)

(b) Write down the equations of the asymptotes to the graph of $y = \frac{4x+1}{2x-1}$. (2 marks)

(c) Sketch the graph of $y = \frac{4x+1}{2x-1}$, indicating clearly each asymptote and also the points of intersection of the graph with the coordinate axes. (3 marks)

(d) Hence, or otherwise, solve the inequality $\frac{4x+1}{2x-1} < -1$. (2 marks)

3 (a) Show that the equation of the normal to the curve $y = x^3 - 7x + 6$ at the point $P(1, 0)$ is $4y - x + 1 = 0$. (4 marks)

(b) The point $R(2k, -k)$ lies on this normal.

Determine the value of k . (2 marks)

(c) Given that Q is the point $(-3, 0)$, find the area of the triangle PQR . (2 marks)

4 (a) Solve the equation $\sec x = 2$ for $0 \leq x \leq 2\pi$. (2 marks)

(b) Use the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$ to show that

$$\cos 2x = 2 \cos^2 x - 1. \quad (2 \text{ marks})$$

(c) Hence solve the equation

$$\cos 2x + 3 \cos x - 1 = 0 \quad \text{for } 0 \leq x \leq 2\pi. \quad (5 \text{ marks})$$

5 (a) Find $\frac{dy}{dx}$ when $y = \frac{e^x}{\sin x}$. (3 marks)

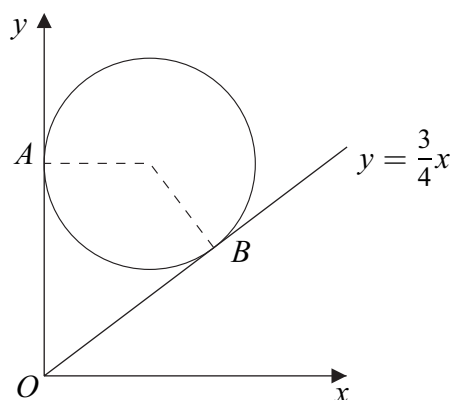
(b) (i) Verify that the graph of $y = \frac{e^x}{\sin x}$ has a stationary point when $x = \frac{\pi}{4}$. (2 marks)

(ii) Find the value of y at this stationary point, giving your answer to three significant figures. (2 marks)

6 The circle C has the equation $(x - 10)^2 + (y - 20)^2 = 100$.

(a) State the radius, and the coordinates of the centre, of the circle C . (2 marks)

(b) The diagram shows the circle C and the line $y = \frac{3}{4}x$ which is the tangent to this circle at the point B . The circle C also touches the y -axis at the point A .



(i) Write down the length of OA and of OB . (1 mark)

(ii) Show that the x -coordinate of B is 16. (2 marks)

(iii) Calculate the length of the chord AB , writing your answer in the form $p\sqrt{5}$, where p is an integer. (3 marks)

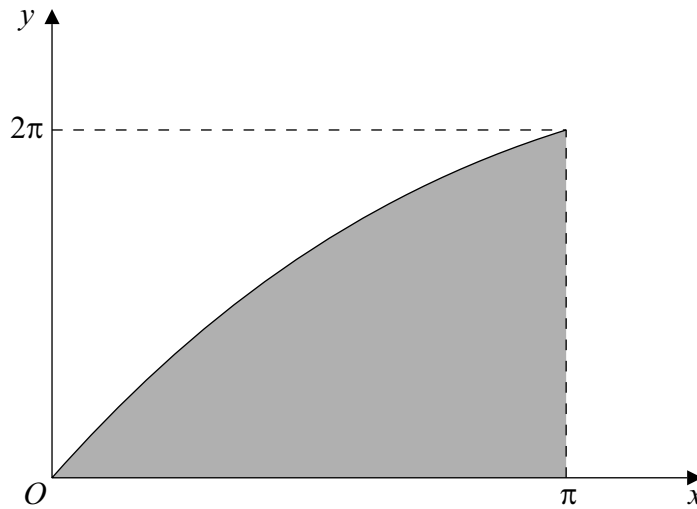
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7 (a) Use integration by parts to show that $\int_0^{\pi} x \sin x \, dx = \pi$. (4 marks)

(b) (i) Express $\sin^2 x$ in terms of $\cos 2x$. (1 mark)

(ii) Hence show that $\int_0^{\pi} \sin^2 x \, dx = \frac{\pi}{2}$. (3 marks)

(c) The diagram below shows the graph of $y = 2x + \sin x$ for $0 \leq x \leq \pi$.



Calculate the volume of revolution of the solid formed when the shaded region is rotated through 2π radians about the x -axis.

Give your answer to three significant figures. (4 marks)

END OF QUESTIONS