GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
(MAP6)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

MAP6

Q	Solution	Marks	Total	Comments
1(a)	$\begin{bmatrix} 4-\lambda & 1 & 1 \\ 0 & 2-\lambda & -6 \\ 0 & -2 & 1-\lambda \end{bmatrix}$			
	$=(4-\lambda)\left[(2-\lambda)(1-\lambda)-12\right]=0$	M1A1		
	$\lambda = 4$	B1		Allow at any stage
	$\lambda = 5, -2$	A1F	4	
(b)	$\begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & -6 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}=4\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$	M1A1		
	$\begin{bmatrix} 4 & 1 & 1 \\ 0 & 2 & -6 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} -5 \\ 18 \\ 12 \end{bmatrix}$			
	$=\begin{bmatrix} 10 \\ -36 \\ 24 \end{bmatrix}=-2\begin{bmatrix} -5 \\ 18 \\ 12 \end{bmatrix}$	A1	3	
(c)	$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -3 & -6 \\ 0 & -2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} -x-y-z \\ -3y-6z \\ -2y-4z \end{bmatrix}$	M1A1F		
	eigenvector $\mathbf{v}_3=\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$	A1	3	OE
(d)	$\mathbf{r}^1 = 4\alpha\mathbf{v}_1 - 2\beta\mathbf{v}_2 + 5\gamma\mathbf{v}_3$	M1A1F	2	
	Total		**12**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\mathbf{a}\times\mathbf{a}+3\mathbf{b}\times\mathbf{a}-\mathbf{a}\times 2\mathbf{b}-3\mathbf{b}\times 2\mathbf{b}$	M1		
	Use of $\mathbf{a}\times\mathbf{a}=0$	B1		or $\mathbf{b}\times\mathbf{b}=0$ PI
	Use of $\mathbf{a}\times\mathbf{b}=-\mathbf{b}\times\mathbf{a}$	B1		PI
	Result $5\,\mathbf{b}\times\mathbf{a}$	A1	4	or $-5\,\mathbf{a}\times\mathbf{b}$
(b)	$\mathbf{a}$ perpendicular $\mathbf{b}\ \lvert\mathbf{a}\times\mathbf{b}\rvert=\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert$	M1		
	Result $5\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert$	A1F	2	
	Total		**6**	
3(a)	$\det\mathbf{A}=a(a-0)-5(4-0)+4(4a-3a)$	M1A1		
	$=a^2+4a-20$	A1F	3	
(b)	$\det\mathbf{AB}=\det\mathbf{B}\Rightarrow\det\mathbf{A}=1$	B1		PI
	$a^2+4a-20=1$	M1		$a^2+4a-20=0$ M0
	$a=-7,\ 3$	A1F	3	
	Total		**6**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\begin{bmatrix}2\\1\\1\end{bmatrix}\times\begin{bmatrix}3\\-2\\1\end{bmatrix}=\begin{bmatrix}3\\1\\-7\end{bmatrix}$	M1A1		If determinant method used allow M1A2, 1, 0
	$\begin{bmatrix}3\\1\\-7\end{bmatrix}\cdot\begin{bmatrix}1\\3\\1\end{bmatrix}=-1$	A1F	3	
(ii)	Sensible reasons e.g.			
	$(\mathbf{u}_1\times\mathbf{u}_2).\mathbf{u}_3\neq 0$			
	$\therefore \mathbf{u}_3\neq\alpha\ \mathbf{u}_1+\beta\mathbf{u}_2$ / not coplanar	B2,1,0	2	
(b)(i)	$\begin{bmatrix}2\\7\\2\end{bmatrix}=\begin{bmatrix}2&3&1\\1&-2&3\\1&1&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}$			Alternative using inverse matrix
				Determinant **M** B1
	One equation with 2 unknowns			Inverse matrix
	e.g. $a+2b=0$	M1A1		$\mathbf{M}^{-1}=\frac{-1}{1}\begin{bmatrix}-5-2&11\\2&1-5\\3&1-7\end{bmatrix}$ M1A2,1,0
	Second equation eg $5a+11b=-1$	A1		$\mathbf{M}^{-1}\begin{bmatrix}2\\7\\2\end{bmatrix}=\begin{bmatrix}2\\-1\\1\end{bmatrix}$ M1A1F
	Solve simultaneously	M1		
	$a=2,\ \ b=-1$	A1F		
	$c=1$	A1F	6	
(ii)	$\begin{bmatrix}2\\7\\2\end{bmatrix}=2\begin{bmatrix}2\\1\\1\end{bmatrix}-1\begin{bmatrix}3\\-2\\1\end{bmatrix}+1\begin{bmatrix}1\\3\\1\end{bmatrix}$	M1A1F	2	
	Total		**13**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)	$\overrightarrow{AB}=\begin{bmatrix}-5\\-3\\3\end{bmatrix}\quad\overrightarrow{AC}=\begin{bmatrix}-2\\-2\\-2\end{bmatrix}$	B1		
	$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{bmatrix}12\\-16\\4\end{bmatrix}$	M1A1	3	ft incorrect $\overrightarrow{AB}$ or $\overrightarrow{AC}$
(ii)	$\Delta ABC=\frac{1}{2}\left\lvert\overrightarrow{AB}\times\overrightarrow{AC}\right\rvert=2\sqrt{26}$	M1A1F	2	OE
(iii)	Π is $12x-16y+4z=-24$			
	$(3x-4y+z=-6)$	M1A1	2	OE
(b)(i)	Equation of l is $\mathbf{r}=\begin{bmatrix}0\\-5\\0\end{bmatrix}+\lambda\begin{bmatrix}3\\-4\\1\end{bmatrix}$	B1	1	OE ft incorrect $\begin{bmatrix}3\\-4\\1\end{bmatrix}$
(ii)	meets Π where			
	$3(3\lambda)-4(-4\lambda-5)+\lambda=-6$	M1		
	$\lambda=-1$	A1F		
	point is $(-3,-1,-1)$	A1F	3	
(c)	height is $\sqrt{(3^2+4^2+1^2)}=\sqrt{26}$	B1		
	Volume of $ABCD$ is $\frac{2\sqrt{26}\sqrt{26}}{3}$	M1		Vol $=\frac{1}{3}\left\lvert\mathbf{a}\times\mathbf{b}\times.\mathbf{c}\right\rvert$ allow B1
	$=\frac{52}{3}$	A1	3	CAO
	Total		**14**	

MAP6 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\cos\theta = -\frac{\sqrt{3}}{2}, \quad \sin\theta = \frac{1}{2}$	M1		Correct answer without adequate working B1
	$\theta = 150^\circ$	A1	2	or $\frac{5\pi}{6}$
(b)	$\cos 2\phi = \frac{1}{2}, \ \sin 2\phi = -\frac{\sqrt{3}}{2}$	M1		
	$2\phi = -60$	A1		or $2\phi = -\frac{\pi}{3}$
	$y = -\tan 30x = -\frac{1}{\sqrt{3}}x$	A1	3	ft simple error: answer must not contain trig functions
(c)(i)	$\mathbf{M}_3 =$ $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$			
	$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	M1A1	2	
(ii)	reflection	B1		
	in $y = x$	B1	2	
	Total		**9**	
	Total		**60**	