General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP5**
Unit Pure 5

Tuesday 18 January 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP5.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^3. *(1 mark)*

(b) Hence find

$$\lim_{x \to 0} \frac{2x\cos x - \sin 2x}{x^3}.$$

(4 marks)

2 The function $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 y + xy^2 + 1.$$

(a) The use of the Euler formula

$$y_{r+1} = y_r + h\mathrm{f}(x_r, y_r)$$

with $y_0 = y(1) = 1$ gives $y_1 = y(1+h) = 1.15$. Determine the value of h that has been used. *(3 marks)*

(b) Show that, with this value of h, use of the same Euler formula gives $y_2 = 1.3328$ correct to four decimal places. *(2 marks)*

(c) Use the formula

$$y_{r+1} = y_{r-1} + 2h\mathrm{f}(x_r, y_r)$$

with the same value of h to find y_3, giving your answer to three decimal places. *(3 marks)*

3 (a) Use integration by parts to show that

$$\int_k^1 \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x = 4(\sqrt{k} - 1) - 2\sqrt{k}\ln k,$$

where $0 < k < 1$. *(5 marks)*

(b) Explain why

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x$$

exists, and find this integral. *(2 marks)*

4 (a) Obtain the roots of the equation

$$m^2 + 4m + 8 = 0,$$

giving your answers in the form $a \pm \mathrm{i}\, b$. *(2 marks)*

(b) Solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 8y = 8\mathrm{e}^{-2x}$$

given that $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$ when $x = 0$. *(12 marks)*

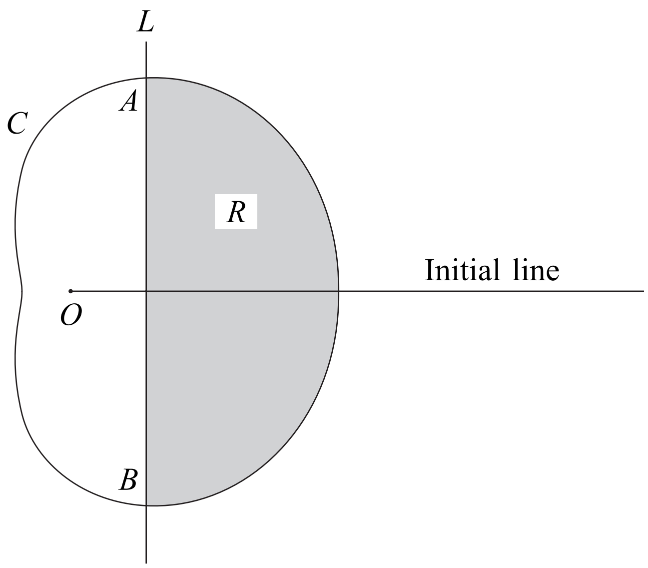
5 The diagram shows a sketch of the curve C and the line L, whose polar equations are

$$r = 3 + 2\cos\theta$$

and

$$r\cos\theta = 2$$

respectively.



(a) The points of intersection of the curve C and the line L are A and B.

Show that $\theta = \frac{1}{3}\pi$ at A and find the polar coordinates of A and B. *(6 marks)*

(b) Find the area of the shaded region R between the curve C and the line L. Give your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational numbers. *(9 marks)*

6 (a) Given that $y = \frac{1}{z}$, where z is a function of x, express $\frac{dy}{dx}$ in terms of z and $\frac{dz}{dx}$. *(1 mark)*

(b) It is given that y satisfies the differential equation

$$x^2 \frac{dy}{dx} + yx = y^2, \quad x > 0 \, .$$

Show that the substitution $y = \frac{1}{z}$ transforms this equation into the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2} \, .$$

(2 marks)

(c) (i) Obtain the general solution of the differential equation

$$\frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2} \, .$$

(5 marks)

(ii) Hence obtain y in terms of x, given that $y = 2$ when $x = \frac{1}{2}$. *(3 marks)*

END OF QUESTIONS