GCE 2005
January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
(MAP5)

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

MAP5

Q	Solution	Marks	Total	Comments
1(a)	$\sin 2x = 2x - \frac{8x^3}{6}$	B1	1	Ignore extra terms
(b)	Use of $\left(1-\frac{x^2}{2}\right)$ and $\left(2x-\frac{8x^3}{6}\right)$	M1		
	$L = \lim_{x\to 0} \frac{2x\left(1-\frac{x^2}{2}\right)-\left(2x-\frac{8x^3}{6}\right)+0\left(x^5\right)}{x^3}$	A1F		Condone $0\left(x^5\right)$ missing
	$= \lim_{x\to 0} \frac{-x^3+\frac{8x^3}{6}+0\left(x^5\right)}{x^3}$	A1F		
	$= \frac{1}{3}$	A1F	4	
	Total		**5**	
2(a)	$1.15 = 1 + h\ (1+1+1)$ $h = 0.05$	M1A1 A1	3	
(b)	$y_2 = 1.15 + .05\,\mathrm{f}(1.05, 1.15)$ $= 1.3328$	M1 A1	2	Applied AG
(c)	$y_3 = 1.15 + 0.1\,\mathrm{f}(1.1, 1.3328)$ $= 1.607$	M1A1 A1	3	2 clear errors M0
	Total		**8**	
3(a)	$\mathrm{I} = \left[2\sqrt{x}\ln x\right]_k^1 - \int_k^1 2\sqrt{x}\frac{1}{x}\,\mathrm{d}x$	M1A1 A1		
	$\left[-4\sqrt{x}\right]_k^1$	A1F		
	$= 4\left(\sqrt{k}-1\right) - 2\sqrt{k}\ln k$	A1	5	AG
(b)	Exists since $\sqrt{k}\ln k \to 0$ as $k \to 0$ value is – 4	E1 A1F	2	Clear explanation If M0 earlier, allow B1 for – 4
	Total		**7**	

MAP5 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$m = \frac{1}{2}\left(-4 \pm \sqrt{16-32}\right)$	M1		
	$= -2 \pm 2\text{i}$	A1	2	
(b)	C.F. $y = \text{e}^{-2x}\left(A\cos 2x + B\sin 2x\right)$	M1A1F		Provided roots are non-real
	P.I. Try $y = P\text{e}^{-2x}$	M1		If P.I. $= A\,x^k\text{e}^{-2x}\left(k \neq 0\right)$ then M0
	$\frac{\text{d}y}{\text{d}x} = -2Pe^{-2x}$; $\frac{\text{d}^2y}{\text{d}x^2} = 4P\text{e}^{-2x}$	A1		A0
	Sub into D.E.	M1		M0
	$4P = 8,\ P = 2$	A1F		A0
	G.S. $y = \text{e}^{-2x}\left(A\cos 2x + B\sin 2x + 2\right)$	B1F		i.e. C.F. + P.I. B1
	$y = 2,\ x = 0 \quad \therefore A = 0$	B1F		B0
	$\frac{\text{d}y}{\text{d}x} = -2B\text{e}^{-2x}\sin 2x + 2B\text{e}^{-2x}\cos 2x - 4\text{e}^{-2x}$	M1A1F		Then follow through last 4 marks
	$2 = 2B - 4 \quad B = 3$	A1F		
	G.S. $y = \left(3\sin^2 x + 2\right)\text{e}^{-2x}$	A1F	12	
	Total		**14**	

MAP5 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\dfrac{2}{\cos\theta} = 3 + 2\cos\theta$	M1		or corresponding results in r
	$2\cos^2\theta + 3\cos\theta - 2 = 0$	A1		
	$(2\cos\theta - 1)(\cos\theta + 2) = 0$	m1		
	$\cos\theta = \dfrac{1}{2}$, $\cos\theta \neq -2$	A1		
	at A, $\theta = \dfrac{\pi}{3}$	A1		
	A and B are $\left(4, \pm\dfrac{1}{3}\pi\right)$	A1	6	Accept $\left(4, \dfrac{5\pi}{3}\right)$
(b)	Area S bounded by C, OA, OB			
	$S = \dfrac{1}{2}\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (3 + 2\cos\theta)^2 \,\mathrm{d}\theta$	M1		Ignore limits here
	$= \displaystyle\int_0^{\frac{\pi}{3}} (9 + 12\cos\theta + 4\cos^2\theta)\,\mathrm{d}\theta$	A1		
	Use of $\cos 2\theta = 2\cos^2\theta - 1$	m1		For an attempt to express $\cos^2\theta$ in terms of $\cos 2\theta$
	$S = \displaystyle\int_0^{\frac{\pi}{3}} (11 + 12\cos\theta + 2\cos 2\theta)\,\mathrm{d}\theta$	A1F		
	$= \left[11\theta + 12\sin\theta + \sin 2\theta\right]_0^{\frac{\pi}{3}}$	A1F		
	$= \dfrac{11\pi}{3} + \dfrac{13\sqrt{3}}{2}$	A1F		Correct limits needed here
	Area of $\Delta OAB = \frac{1}{2}4^2\sqrt{\frac{3}{2}} = 4\sqrt{3}$	M1A1		Allow M1 for $\left[2\tan\theta\right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$
	Area of $R = \dfrac{11\pi}{3} + \dfrac{13\sqrt{3}}{2} - 4\sqrt{3}$			
	$= \dfrac{11\pi}{3} + \dfrac{5\sqrt{3}}{2}$	A1F	9	
	Total		**15**	

MAP5 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$y=\frac{1}{z},\ \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{z^2}\frac{\mathrm{d}z}{\mathrm{d}x}$	B1	1	
(b)	$x^2\left(-\frac{1}{z^2}\frac{\mathrm{d}z}{\mathrm{d}x}\right)+\frac{1}{z}x=\frac{1}{z^2}$	M1		
	$\frac{\mathrm{d}z}{\mathrm{d}x}-\frac{z}{x}=-\frac{1}{x^2}$	A1	2	
(c)(i)	$\text{I.F}=\mathrm{e}^{-\int\frac{1}{x}\mathrm{d}x}=\frac{1}{x}$	M1A1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{z}{x}\right)=-\frac{1}{x^3}$	M1A1F		
	$\frac{z}{x}=\frac{1}{2x^2}+c$	A1F	5	
(ii)	$x=\frac{1}{2},\ z=\frac{1}{2}\quad c=-1$	A1F		
	$z=\frac{x\left(1-2x^2\right)}{2x^2}$	A1F		
	$y=\frac{2x}{1-2x^2}$	A1F	3	Accept $\left(\frac{1}{2x}-x\right)^{-1}$ OE
	Total		**11**	
	Total		**60**	