GCE 2005

January Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A

(MAP4)

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Key to Mark Scheme

M .... mark is for ....method
m.................................... mark is dependent on one or more M marks and is for ....method
A .... mark is dependent on M or m marks and is for................................accuracy
B .... mark is independent of M or m marks and is for ....method and accuracy
E .... mark is for .... explanation
√ or ft or F ....follow through from previous incorrect result
CAO .... correct answer only
AWFW... anything which falls within
AWRT... anything which rounds to
AG ....answer given
SC..special case
OE ....or equivalent
A2,1 ....2 or 1 (or 0) accuracy marks
-*x* EE ....deduct *x* marks for each error
NMS ....no method shown
PI .... possibly implied
SCA ....substantially correct approach
c ....candidate
SF .... significant figure(s)
DP...decimal place(s)

Abbreviations used in Marking

MC – *x* .... deducted *x* marks for mis-copy
MR – *x* .... deducted *x* marks for mis-read
ISW ....ignored subsequent working
BOD ....given benefit of doubt
WR ....work replaced by candidate
FB..formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working .... mark as in scheme
Incorrect answer without working... zero marks unless specified otherwise

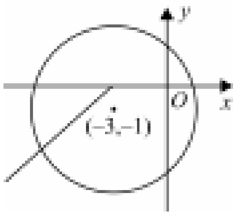
More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only
Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

MAP4

Q	Solution	Marks	Total	Comments
1(a)	$\beta = 1 - 2\mathrm{i}$	B1	1	
(b)(i)	$\alpha\beta = (1+2\mathrm{i})(1-2\mathrm{i}) = 5$	B1	1	
(ii)	$\alpha\beta\gamma = -30 \quad \gamma = -6$	M1A1F	2	
(c)	Method for either p or q	M1		
	$p = 4, \quad q = -7$	A1FA1F	3	
	Total		**7**	
2(a)	LHS = $r^2\left(r^2 + 2r + 1 - \left(r^2 - 2r + 1\right)\right)$	M1		
	$= 4r^3$	A1	2	AG
(b)	$4 \times 50^3 = 50^2 \times 51^2 - 49^2 \times 50^2$			For $100^2 \times 101^2 - 50^2 \times 51^2$ M1A0m1A0
	$4 \times 51^3 = 51^2 \times 52^2 - 50^2 \times 51^2$			For $100^2 \times 99^2 - 49^2 \times 50^2$ M1A0m1A0
	$4 \times 100^3 = 100^2 \times 101^2 - 99^2 \times 100^2$	M1A1		
	$4S = 100^2 \times 101^2 - 49^2 \times 50^2$	m1		Clear cancellation shown.
	$S = 24001875$	A1F	4	If $\sum r^3$ quoted mark M1A1 only
	Total		**6**	
3(a)	$r = \sqrt{2}, \quad \theta = \frac{1}{4}\pi$	B1B1	2	
(b)	$(1+\mathrm{i})^{21} - (1-\mathrm{i})^{21}$			
	$= \left(\sqrt{2}\right)^{21} \mathrm{e}^{\frac{21\pi\mathrm{i}}{4}} - \left(\sqrt{2}\right)^{21} \mathrm{e}^{\frac{-21\pi\mathrm{i}}{4}}$	M1A1		
	$\left(\sqrt{2}\right)^{21}\left(\cos\frac{21\pi}{4} + \mathrm{i}\sin\frac{21\pi}{4}\right.$ $\left. - \cos\frac{21\pi}{4} + \mathrm{i}\sin\frac{21\pi}{4}\right)$	A1F		
	$= \left(\sqrt{2}\right)^{21} 2\mathrm{i}\sin\frac{21\pi}{4}$	A1F		If $\sqrt{2}$ not $\left(\sqrt{2}\right)^{21}$ lose final A1 also
	$= -2048\mathrm{i}$	A1F	5	provided of the correct form
	Total		**7**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)				
	Circle	B1		
	Centre below x – axis, radius $\simeq 5$	B1	2	
(ii)	Half line	B1		
	with gradient ≈ 1 through $(-3,0)$	B1	2	
(b)(i)	Explanation from diagram	B1	1	
(ii)	Verification that $\lvert -7-4\text{i}+3+\text{i}\rvert = 5$	M1A1		
	Verification that arg $(-7-4\text{i}+3) = -\dfrac{3\pi}{4}$	M1A1	4	
	Total		**9**	
5(a)(i)	$P\left(\dfrac{\text{e}^{x}+\text{e}^{-x}}{2}\right)+q\left(\dfrac{\text{e}^{x}-\text{e}^{-x}}{2}\right)=r$	M1		
	$(p+q)\text{e}^{x}+(p-q)\text{e}^{-x}=2r$	A1		
	$(p+q)\text{e}^{2x}-2r\text{e}^{x}+(p-q)=0$	A1	3	AG
(ii)	$\text{e}^{x}=\dfrac{2r\pm\sqrt{4r^{2}-4(p-q)(p+q)}}{2(p+q)}$	M1A1		b^2-4ac <u>only</u> used M1A1 only
	Use of $p^2=q^2+r^2$ to show that			
	$\text{e}^{x}=\dfrac{r}{p+q}$	m1A1		
	$\text{e}^{x}>0 \Rightarrow$ one solution	E1	5	
(b)	$\text{e}^{x}=\dfrac{12}{18}$	M1		
	$x=\ln\left(\dfrac{2}{3}\right)$	A1	2	CAO
	Total		**10**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$\mathrm{f}(n+1)-\mathrm{f}(n)=4\times7^{n+1}+3\times5^{n+1}$			
	$+5-4\times7^{n}-3\times5^{n}\ -5$	M1		
	Grouping in powers of 7 and 5	m1		
	$=4\times7^{n}(7-1)+3\times5^{n}(5-1)$	A1		
	$=24\times7^{n}+12\times5^{n}$	A1	4	AG
(b)	$\mathrm{f}(1)=\mathrm{M}(12)$ shown	B1		
	Assume result true for $n=k$			
	Then $\mathrm{f}(k+1)=\mathrm{f}(k)+\mathrm{M}(12)$	M1		
	$=\mathrm{M}(12)$	A1		Clear demonstration
	$\mathrm{P}(k)\Rightarrow\mathrm{P}(k+1)$ and $\mathrm{P}(1)$ true	E1	4	Provided M1 earned
	Total		**8**	

MAP4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}}$	B1		
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x\sqrt{1+x^2}\right) = \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}}$	M1A1		Reasonable attempt at product rule for M1
	Result $= 2\sqrt{1+x^2}$	A1	4	AG
(b)(i)	$S = 2\pi \int_{\ln\left(\frac{3}{4}\right)}^{\ln\left(\frac{4}{3}\right)} \mathrm{e}^x \sqrt{1+\left(\mathrm{e}^x\right)^2}\,\mathrm{d}x$	M1		
	$=2\pi \int_{\ln\left(\frac{3}{4}\right)}^{\ln\left(\frac{4}{3}\right)} \mathrm{e}^x \sqrt{1+\mathrm{e}^{2x}}\,\mathrm{d}x$	A1	2	AG
(ii)	$u = \mathrm{e}^x, \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$	M1		Use of formula possibly implied
	$S = 2\pi \int_{\frac{3}{4}}^{\frac{4}{3}} \sqrt{1+u^2}\,\mathrm{d}u$	A1		Must be of this from to score further marks ignore limits here
	$=\pi\left[\sinh^{-1} u + u\sqrt{1+u^2}\right]_{\frac{3}{4}}^{\frac{4}{3}}$	A1		
	$=\left[\sinh^{-1}\frac{4}{3} + \frac{4}{3}\sqrt{1+\left(\frac{4}{3}\right)^2}\right.$	A1F		
	$\left.-\sinh^{-1}\frac{3}{4} - \frac{3}{4}\sqrt{1+\left(\frac{3}{4}\right)^2}\right]$			
	$=\pi\left[\ln 3 + \frac{20}{9} - \ln 2 - \frac{15}{16}\right]$	m1A1F		
	$=\pi\left[\ln\frac{3}{2} + \frac{185}{144}\right]$	A1F	7	
	Total		**13**	
	Total		**60**	