General Certificate of Education
January 2005
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP3**
Unit Pure 3

Thursday 27 January 2005 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve is defined by the parametric equations

$$x = \sin t, \qquad y = 2\cos t.$$

(a) Find the coordinates of the point P on the curve for which $t = \frac{\pi}{3}$. *(1 mark)*

(b) (i) Find $\frac{dy}{dx}$ in terms of t. *(2 marks)*

(ii) Hence find the gradient of the curve at P. *(1 mark)*

(c) Find the equation of the tangent to the curve at P, giving your answer in the form $y = px + q$. *(2 marks)*

2 (a) (i) Obtain the first **four** terms in the binomial expansion of $\frac{1}{1+x}$ in ascending powers of x. *(2 marks)*

(ii) Show that the first four terms in the binomial expansion of $\frac{1}{3+2x}$ in ascending powers of x are

$$\frac{1}{3} - \frac{2}{9}x + \frac{4}{27}x^2 - \frac{8}{81}x^3.$$ *(3 marks)*

(b) Express $\frac{8+7x}{(1+x)(3+2x)}$ in the form $\frac{A}{1+x} + \frac{B}{3+2x}$. *(3 marks)*

(c) Hence obtain the first **four** terms in the expansion of $\frac{8+7x}{(1+x)(3+2x)}$ in ascending powers of x. *(3 marks)*

3 (a) On 1 January 1998, Company A issued some shares at a price of 90 p each. Investors expected the shares to increase in value according to the model

$$P = 90 \times 1.12^{t},$$

where the value of each share after t years is P pence.

Show that, according to this model, the value of each share on 1 January 2005, correct to the nearest penny, is 199 p. *(2 marks)*

(b) On 1 January 1998, Company B issued some shares at a price of 270 p each. On 1 January 2005, these shares are worth 405 p each. The value, Q pence, of each share t years after their issue can be represented by the model

$$Q = 270 \times k^{t},$$

where k is a constant.

Show that the value of k, correct to two decimal places, is 1.06 . *(3 marks)*

(c) Assuming that the two models remain valid, there is a time at which a share in Company A will be equal in value to a share in Company B.

Determine the year in which this happens. *(4 marks)*

4 (a) The function f is given by $f(x) = e^{-3x}$.

(i) Find $f'(x)$ and $f''(x)$. *(2 marks)*

(ii) Hence show that the first three terms of the Maclaurin series of $f(x)$ are

$$1 - 3x + \frac{9}{2}x^2.$$ *(2 marks)*

(b) Use the approximation $\ln(1 + x) \approx x - \frac{x^2}{2} + \frac{x^3}{3}$ to obtain a similar approximation for $\ln(1 + 3x)$ for small values of x. *(2 marks)*

(c) Use the results for parts (a) and (b) to estimate the positive value of x for which

$$\ln(1 + 3x) - 2xe^{-3x} = 0.1 .$$

Give your value of x to three decimal places. *(4 marks)*

5 Karen is designing a light show. She wants circles of light to appear on a big screen and to increase in size. In her design, a circle will initially have a radius of 50 cm, which will increase to 250 cm in 5 seconds. The radius of a circle is r cm at time t seconds. Karen considers two possible designs, A and B.

(a) **Design A** The rate of increase of the radius is constant.

Find the radius of the circle when $t = 2$. *(2 marks)*

(b) **Design B** Karen wants the rate at which the radius increases to slow down as t increases. She bases this design on the model $\dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{k}{r}$, where k is a constant.

(i) Solve this differential equation and show that $k = 6000$. *(4 marks)*

(ii) Find the radius of the circle when $t = 2$. *(1 mark)*

(iii) Show that, in Design B, the rate at which the area of the circle increases is constant. *(3 marks)*

6 The line l passes through the points A (3, 5, 1) and B (5, 2, −1).
The plane Π has equation $2x + y - 3z = 1$.

(a) Find a vector equation of the line l. *(2 marks)*

(b) The point C (1, 8, 3) is the point of intersection of l and Π. Verify that C lies on both l and Π. *(3 marks)*

(c) The point D on Π is such that angle ADC is a right angle.

(i) Find an equation for the line AD in the form $\mathbf{r} = \mathbf{a} + t\mathbf{p}$. *(2 marks)*

(ii) Hence show that the coordinates of D are (2, 4.5, 2.5). *(3 marks)*

(iii) Calculate the angle CAD. *(4 marks)*

END OF QUESTIONS