

General Certificate of Education
January 2005
Advanced Subsidiary Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 1

MAP1

Friday 14 January 2005 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Consider the arithmetic series

$$101 + 104 + 107 + 110 + \dots + 800.$$

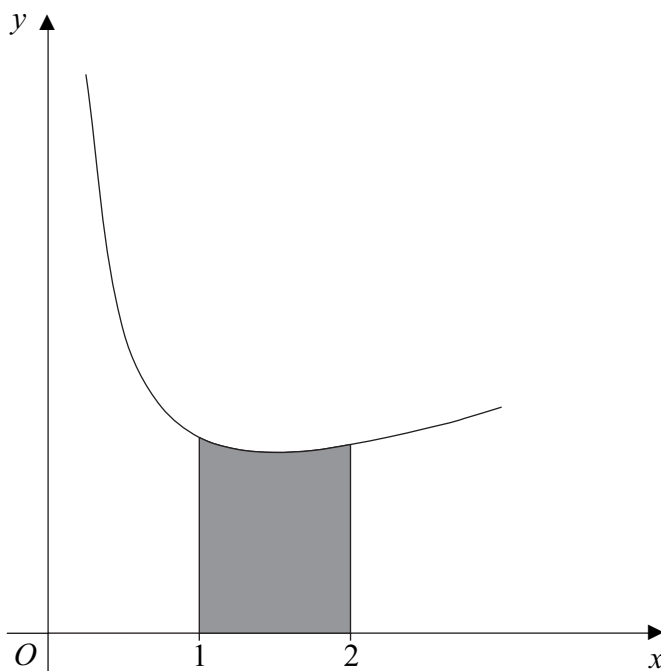
- (a) Show that there are 234 terms in this series. *(2 marks)*
- (b) Find the sum of the arithmetic series. *(3 marks)*
- (c) Find the sum of the **even** numbers in the series. *(2 marks)*

2 A curve has equation

$$y = 4x + 9x^{-1}.$$

- (a) (i) Find $\frac{dy}{dx}$. *(3 marks)*
- (ii) Hence find the coordinates of the **two** stationary points on the curve. *(5 marks)*
- (b) The diagram shows the graph of

$$y = 4x + \frac{9}{x} \quad \text{for } x > 0.$$



- (i) Find $\int \left(4x + \frac{9}{x}\right) dx$. *(2 marks)*
- (ii) Hence calculate the area of the region shaded on the diagram. Give your answer in the form $p + q \ln 2$. *(3 marks)*

3 (a) Write down the values of $\tan \frac{\pi}{4}$ and $\tan \frac{3\pi}{4}$. (2 marks)

(b) It is given that x satisfies the equation

$$2 \sin^2 x + \sin x \cos x = \cos^2 x.$$

(i) By dividing both sides of this equation by $\cos^2 x$, show that

$$2 \tan^2 x + \tan x - 1 = 0. \quad (2 \text{ marks})$$

(ii) Solve this quadratic equation for $\tan x$. (2 marks)

(iii) Hence find all possible values of x in the interval $0 < x < \pi$. Give each answer to three significant figures. (2 marks)

4 (a) Sketch the graph of $y = \ln x$, showing the coordinates of the point where the graph intersects the x -axis. (2 marks)

(b) For the graph of $y = 1 + \ln x$, find:

(i) $\frac{dy}{dx}$; (1 mark)

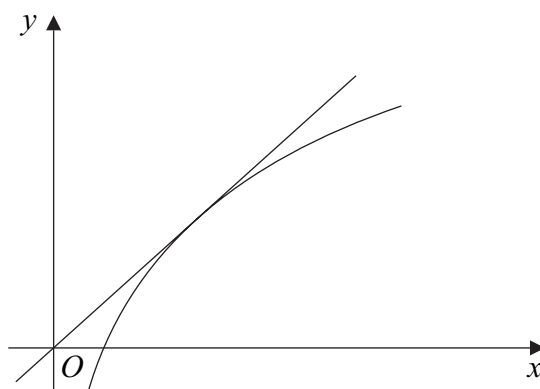
(ii) the gradient of the graph at the point where $x = 1$. (1 mark)

(c) The function f is defined for $x > 0$ by

$$f(x) = 1 + \ln x.$$

The diagram shows the graphs of

$$y = x \text{ and } y = f(x).$$

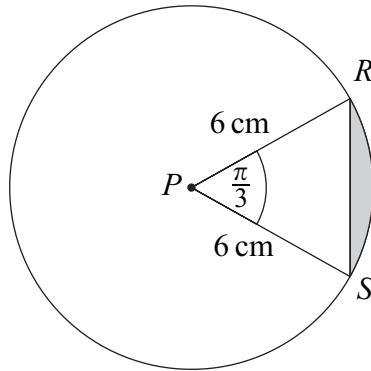


(i) Copy the diagram and, using the same axes, sketch the graph of $y = f^{-1}(x)$. (2 marks)

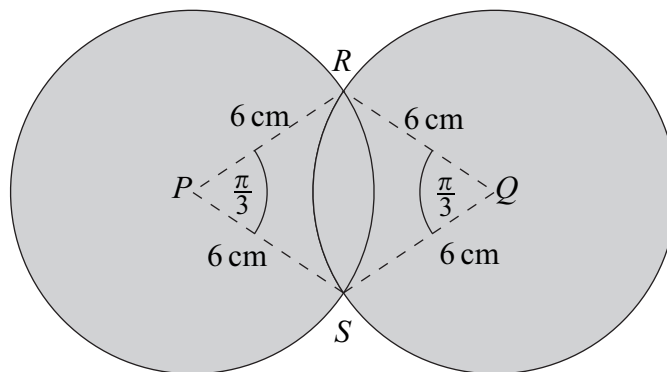
(ii) Find an expression for $f^{-1}(x)$. (3 marks)

Turn over ►

- 5 The diagram shows a circle with centre P and radius 6 cm. The angle RPS is $\frac{\pi}{3}$ radians.



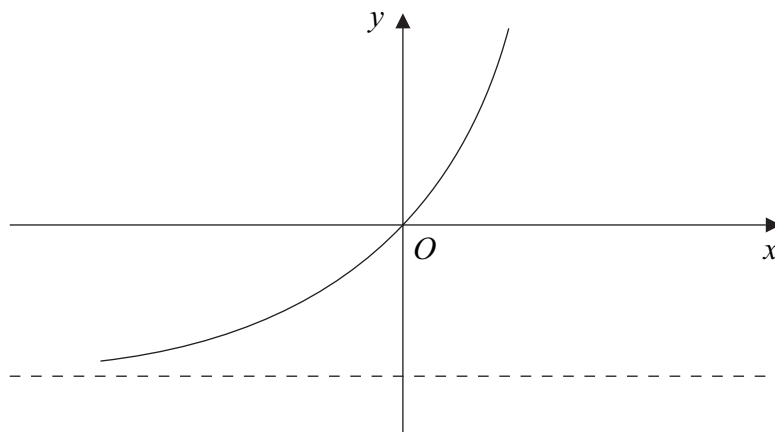
- (a) Find, in terms of π , the length of the arc RS . (2 marks)
- (b) Find the area of:
- (i) the sector PRS ; (2 marks)
 - (ii) the triangle PRS ; (2 marks)
 - (iii) the shaded segment. (1 mark)
- (c) The diagram below shows a logo consisting of two overlapping circles with centres P and Q and both with radius 6 cm. The circles intersect at R and S , and the angle RPS is $\frac{\pi}{3}$ radians.



Show that the area of the front face of the logo is approximately 220 cm^2 . (3 marks)

6 The diagram shows the graph of $y = f(x)$, where the function f is defined for all values of x by

$$f(x) = e^{2x} - 1.$$



(a) (i) Describe a sequence of **two** transformations by which the graph of $y = e^{2x} - 1$ can be obtained from that of $y = e^x$. (4 marks)

(ii) Write down the range of the function f . (1 mark)

(b) For the graph of $y = e^{2x} - 1$, find:

(i) $\frac{dy}{dx}$; (2 marks)

(ii) $\frac{d^2y}{dx^2}$. (1 mark)

(c) The function g is defined for all values of x by

$$g(x) = |x|.$$

(i) Write down an expression for $gf(x)$. (1 mark)

(ii) Sketch the graph of $y = gf(x)$. (2 marks)

(iii) Show that $gf(x) > 1 \Rightarrow f(x) > 1$. (2 marks)

END OF QUESTIONS

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