General Certificate of Education
January 2005
Advanced Level Examination

AQA

ASSESSMENT and
QUALIFICATIONS
ALLIANCE

MATHEMATICS (SPECIFICATION A)   **MAD2**
Unit Discrete 2

Thursday 27 January 2005   Afternoon Session

In addition to this paper you will require:
   • an 8-page answer book;
   • an insert for use in Questions 2 and 4 (enclosed);
   • the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
• Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
• Write the information required on the front of your answer book. The *Examining Body* for this
  paper is AQA. The *Paper Reference* is MAD2.
• Answer **all** questions.
• All necessary working should be shown; otherwise marks for method may be lost.
• The **final** answer to questions requiring the use of tables or calculators should normally be given to
  three significant figures.
• Tie loosely any additional sheets you have used, including the insert for use in Questions 2 and 4, to
  the back of your answer book before handing it to the invigilator.

Information
• The maximum mark for this paper is 60.
• Mark allocations are shown in brackets.
• Further copies of the insert for use in Questions 2 and 4 are available on request.

Advice
• Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MAD2

Answer **all** questions.

1 An office manager has four secretaries available and four tasks to be completed. Each secretary is to be assigned a task. The time, in minutes, required by each secretary to complete each task is given in the table.

	Task 1	Task 2	Task 3	Task 4
A	23	27	21	26
B	28	28	26	27
C	24	25	23	24
D	26	24	23	21

Use the Hungarian algorithm to obtain the allocation of secretaries to tasks that minimises the total time. State this minimum time.   *(6 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

The following diagram shows an activity network for a major project.
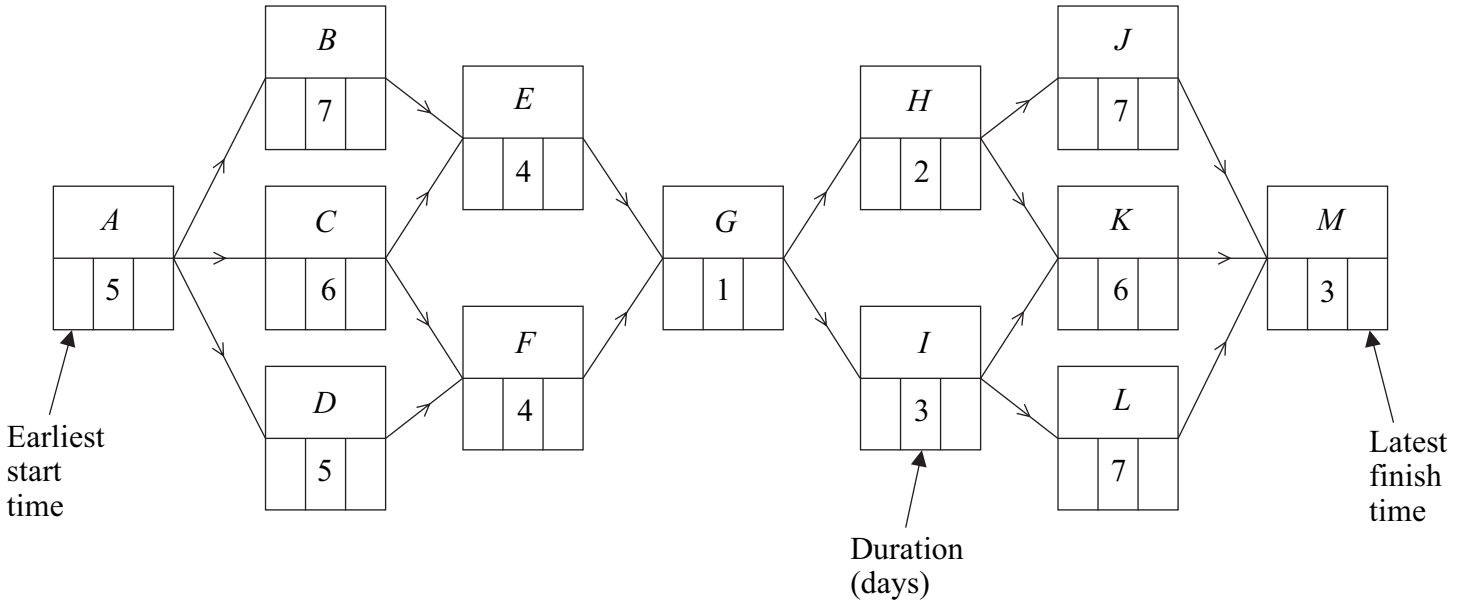
(a)   On **Figure 1**:

   (i)   find the earliest start time for each activity;   *(2 marks)*

   (ii)   find the latest finish time for each activity.   *(2 marks)*

(b)   Identify the critical path.   *(1 mark)*

(c)   Write down the activity that has a float time of more than 1 day.   *(1 mark)*

(d)   Given that activity *C* overruns by 3 days, find the overrun time for the whole project.
                                                                            (2 marks)

(e)   Given that activities *C* and *F* overrun by 3 days each, find the overrun time for the whole project.   *(3 marks)*

3 Cara is to fly from Manchester to Palermo. The journey has three legs.

Leg 1 is from Manchester to Stansted, Gatwick or Luton.
Leg 2 is from one of these airports to Turin, Rome or Venice.
Leg 3 is from Turin, Rome or Venice to Palermo.

Her travelling time consists of flying time and changeover time.

Figure 2 shows travelling times between airports and **Figure 3** shows changeover times. All times are in minutes.

	Manchester (M)	Stansted (S)	Gatwick (G)	Luton (L)	Turin (T)	Rome (R)	Venice (V)	Palermo (P)
Manchester (M)	—	50	45	40	—	—	—	—
Stansted (S)	50	—	—	—	52	60	48	—
Gatwick (G)	45	—	—	—	55	65	60	—
Luton (L)	40	—	—	—	70	75	65	—
Turin (T)	—	52	55	70	—	—	—	55
Rome (R)	—	60	65	75	—	—	—	40
Venice (V)	—	48	60	65	—	—	—	60
Palermo (P)	—	—	—	—	55	40	60	—

Figure 2

|	**Turin** (*T*)	**Rome** (*R*)	**Venice** (*V*)	**Palermo** (*P*)
Stansted (*S*)	75	70	75	—
Gatwick (*G*)	75	70	70	—
Luton (*L*)	70	64	70	—
Turin (*T*)	—	—	—	70
Rome (*R*)	—	—	—	80
Venice (*V*)	—	—	—	65

Figure 3

(a) Show that the total time taken for Cara to travel from Manchester to Palermo via Gatwick and Rome is 300 minutes. *(2 marks)*

(b) By drawing a network diagram, or otherwise, use dynamic programming to find the minimum time for Cara to travel from Manchester to Palermo. State the corresponding route. *(9 marks)*

TURN OVER FOR THE NEXT QUESTION

4 [**Figure 4**, printed on the insert, is provided for use in answering this question.]

The following diagram shows the numbers of pupils who can pass along various corridors of a school building in one minute in the event of a fire drill. The classroom areas are A, B and C. The fire exits are O, P and Q.
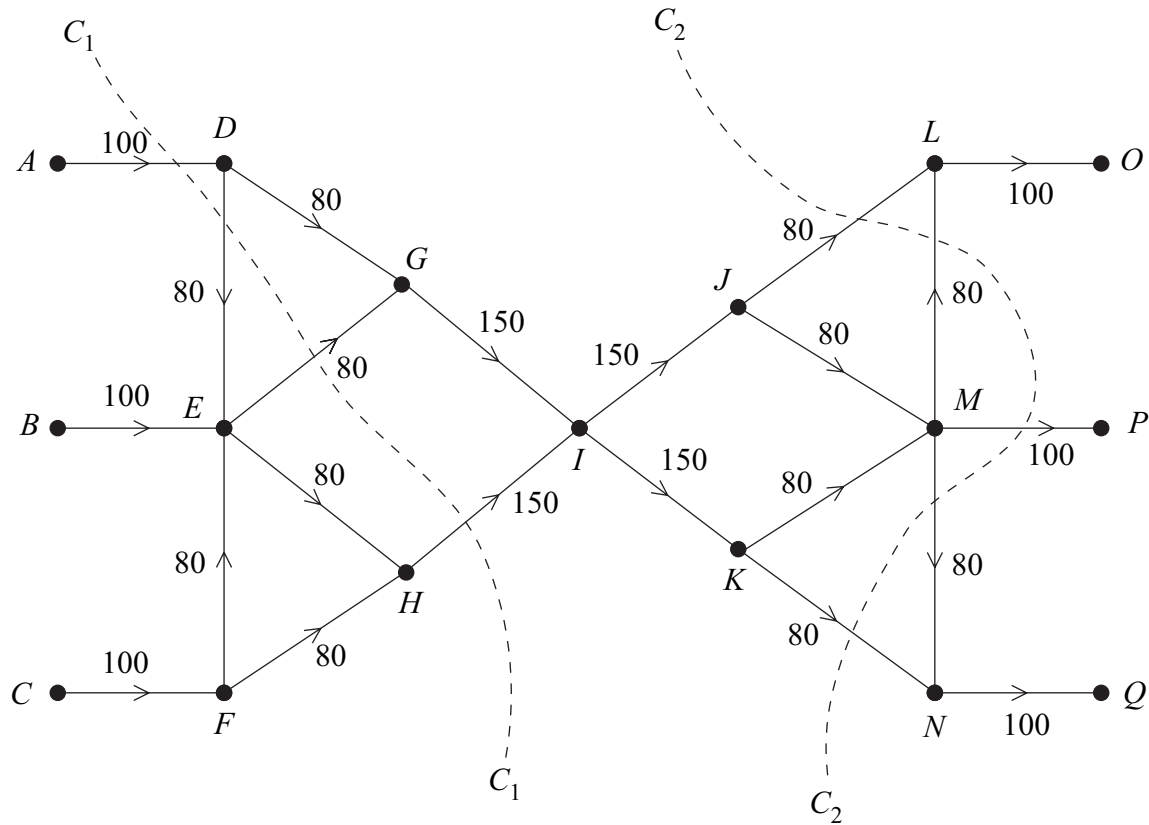
(a) Find the values of the cuts C_1 and C_2. *(2 marks)*

(b) On **Figure 4**, show a flow of 300 pupils per minute through the network. *(3 marks)*

(c) (i) On a certain day there is a restriction at vertex I of 120 pupils per minute.

Find the maximum flow on this day. *(1 mark)*

(ii) On another day the junction at I is clear but there is a restriction at vertex H of 120 pupils per minute.

Find the maximum flow on this day. *(2 marks)*

5 (a) Display the following linear programming problem in a Simplex tableau.

$$\text{Maximise} \quad P = 2x + 3y$$
$$\text{subject to} \quad 2x + y \leqslant 20$$
$$x + 2y \leqslant 20$$
$$5x + 4y \leqslant 60 \qquad \qquad \textit{(2 marks)}$$

(b) Solve the problem using the Simplex algorithm, giving your answers as exact fractions.
(9 marks)

6 Two people, *A* and *B*, play a zero-sum game. The game is represented by the following pay-off matrix for *A*.

		B		
	Strategy	**I**	**II**	**III**
	I	4	−1	2
A	**II**	2	3	1
	III	1	−2	1

(a) Explain why it will never be optimal for *A* to adopt strategy III. *(1 mark)*

(b) (i) Find the optimal mixed strategy for *A*. *(7 marks)*

(ii) Show that the value of the game is $\frac{7}{5}$. *(1 mark)*

(c) By considering optimal mixed strategies for *B*, show that player *B* should play strategy II with a probability of $\frac{1}{5}$. *(4 marks)*

END OF QUESTIONS

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Insert for use in answering Questions 2 and 4.

Fill in the boxes at the top of this page.

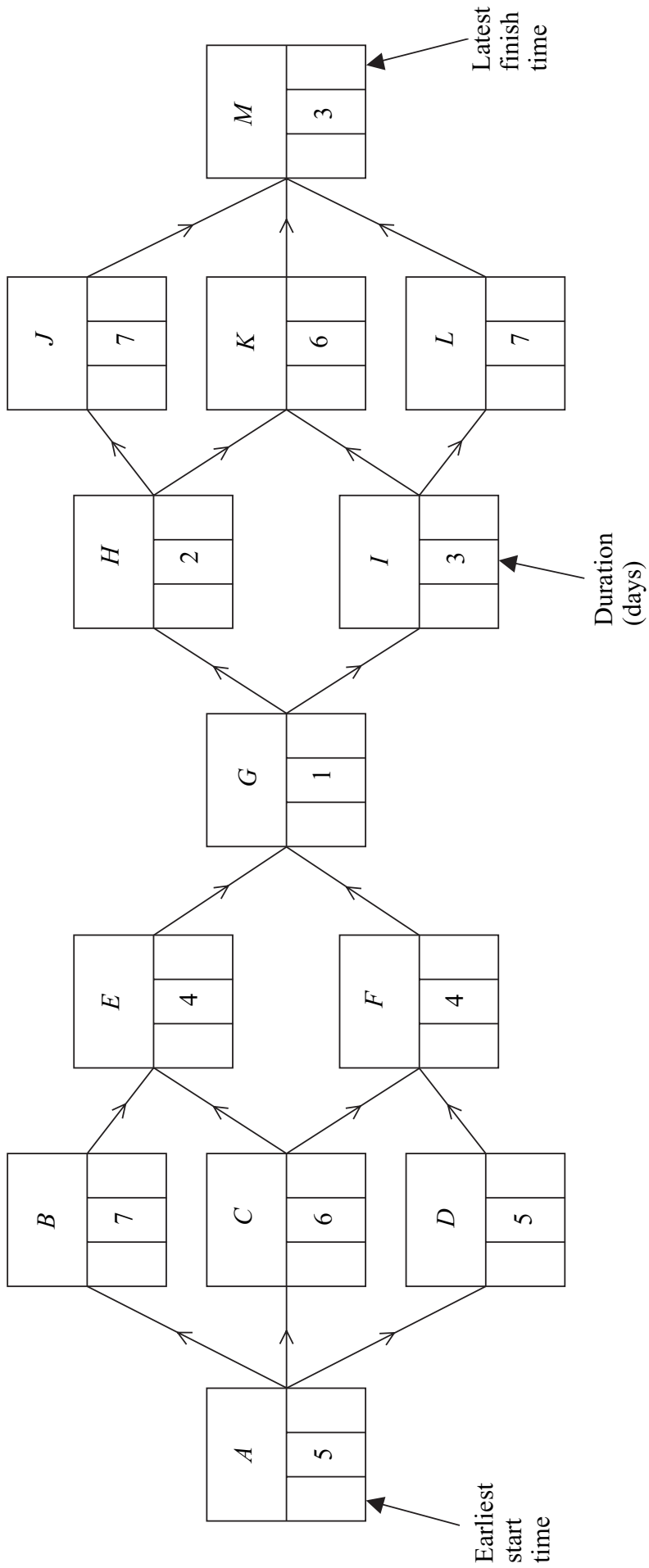Fasten this insert securely to your answer book.
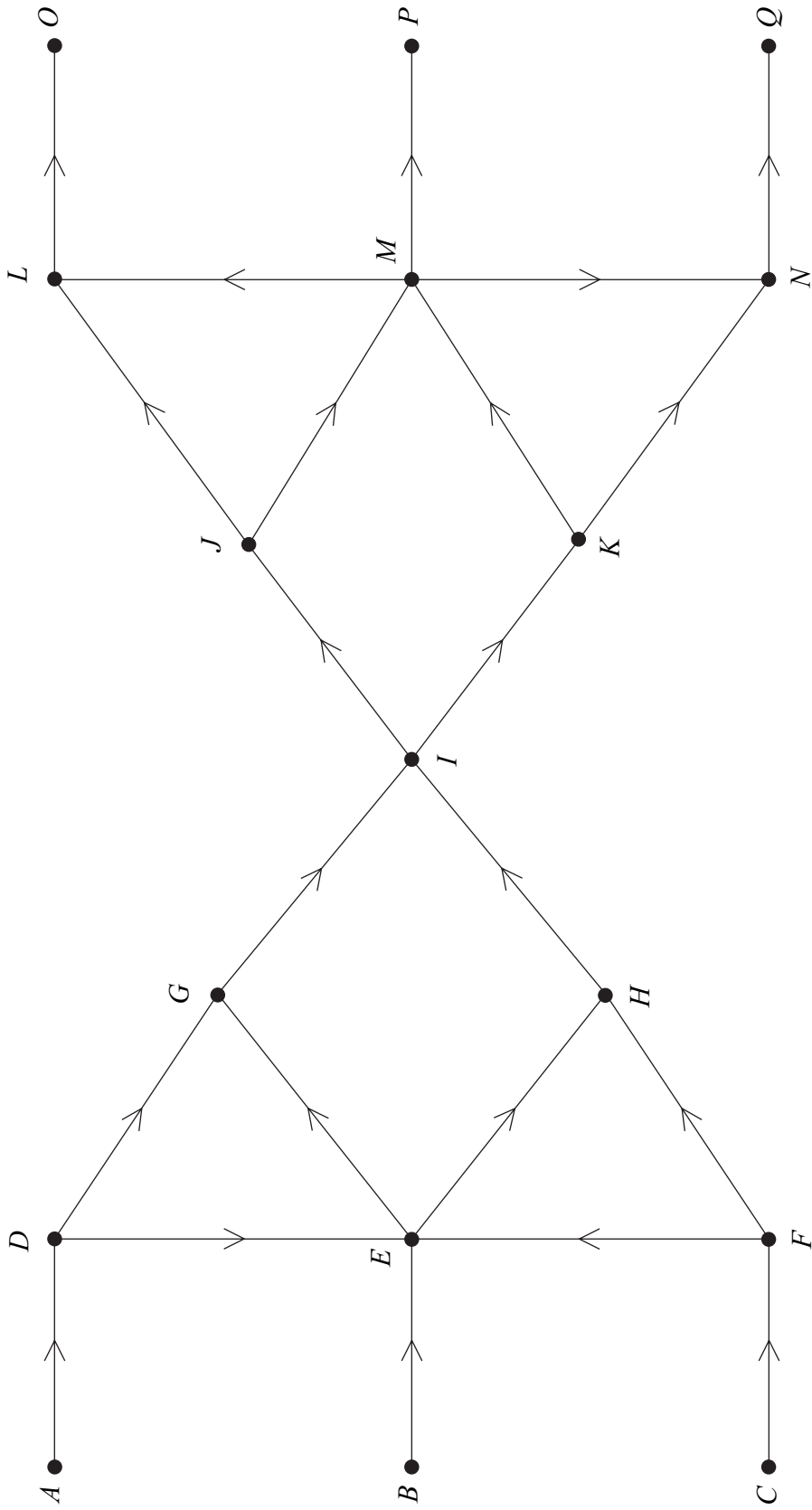
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Figure 1 (for Question 2)

Figure 4 (for Question 4)

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