

General Certificate of Education
November 2004
Advanced Subsidiary Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 1

MAP1

Tuesday 2 November 2004 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- a ruler;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

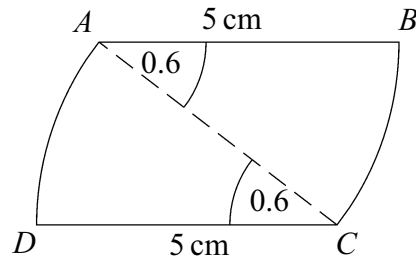
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 The diagram shows a shape $ABCD$ consisting of two sectors of circles joined along the side AC . The centres of the circles are A and C . Each sector has radius 5 cm and angle 0.6 radians.

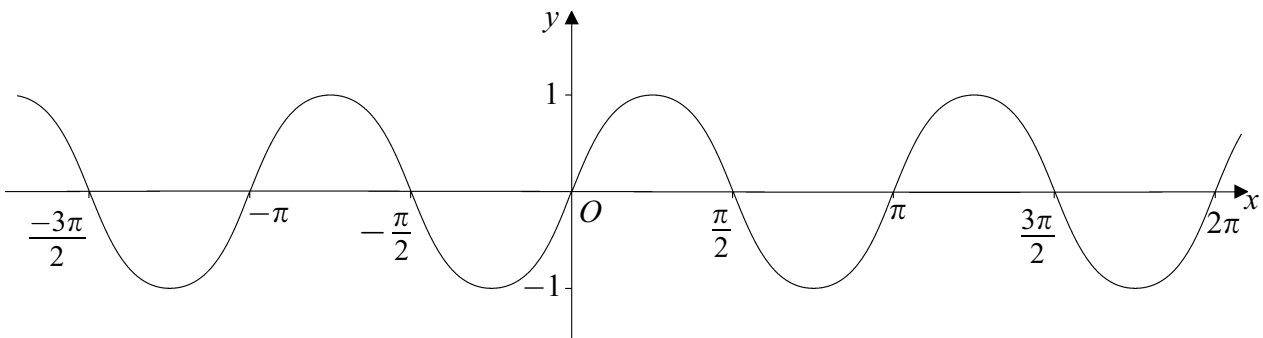


Calculate:

- (a) the area of the shape $ABCD$; (2 marks)
- (b) the perimeter of the shape $ABCD$. (3 marks)

- 2 The diagram shows the graph of $y = f(x)$, where

$$f(x) = \sin 2x.$$



- (a) (i) State, giving a reason, whether the function f is odd, even or neither. (2 marks)
- (ii) State the period of the function f . (1 mark)
- (b) (i) The graph of $y = f(x)$ is stretched parallel to the y -axis with scale factor 3. Write down the equation of the resulting graph. (2 marks)
- (ii) The graph obtained in part (b)(i) is now translated by $\frac{\pi}{2}$ units in the positive x direction.

Write down the equation of the resulting graph. (2 marks)

3 The function f is defined for **positive** values of x by

$$f(x) = x + 100 + \frac{25}{x}.$$

(a) (i) Given that $y = f(x)$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (4 marks)

(ii) Find the coordinates of the stationary point on the graph of $y = f(x)$. (3 marks)

(iii) Find the value of $\frac{d^2y}{dx^2}$ at the stationary point and hence determine whether this point is a maximum or a minimum. (2 marks)

(b) A factory manager is planning the manufacture of a fixed number of items, divided into equal batches, in the coming year. The cost, in hundreds of pounds, of manufacturing x batches is $f(x)$, where f is the function defined above.

State the value of x which corresponds to the minimum cost and find the minimum cost. (2 marks)

4 A geometric sequence u_1, u_2, u_3, \dots is defined by

$$u_n = 2 \times 4^n.$$

(a) (i) Write down the values of u_1 and u_2 , and verify that $u_3 = 128$. (2 marks)

(ii) State the common ratio. (1 mark)

(iii) Show that the sum of the first 20 terms is $\frac{8}{3}(4^{20} - 1)$. (3 marks)

(b) Another sequence v_1, v_2, v_3, \dots is defined by

$$v_n = \log_2(u_n).$$

(i) Show that $v_1 = 3$. (1 mark)

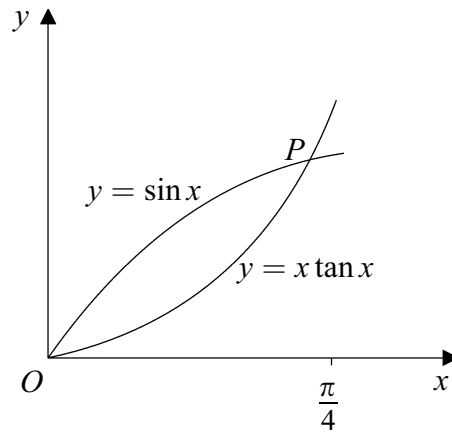
(ii) By simplifying $\log_2(2 \times 4^n)$, show that

$$v_n = 1 + 2n. \quad (3 \text{ marks})$$

5 The diagram shows the graphs of

$$y = \sin x \quad \text{and} \quad y = x \tan x, \quad 0 \leq x \leq \frac{\pi}{4},$$

intersecting at the origin and at the point P .



(a) Show that, at the points of intersection,

$$\sin x(\cos x - x) = 0. \quad (3 \text{ marks})$$

(b) Explain why, at the point P ,

$$\cos x - x = 0. \quad (1 \text{ mark})$$

(c) State, giving a reason, whether

$$\cos x - x = 0$$

is a **necessary** condition for the equation in part (a) to be satisfied. (2 marks)

(d) (i) Show that the equation

$$\cos x - x = 0$$

has a root between 0.7 and 0.8. (3 marks)

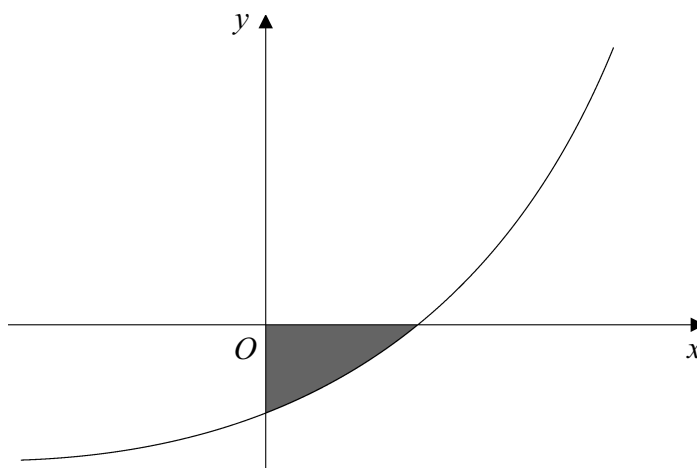
(ii) Use interval bisection to determine whether the root is closer to 0.7 or to 0.8.

(2 marks)

- 6 The function f is defined for all values of x by

$$f(x) = e^x - 3.$$

The diagram shows the graph of $y = f(x)$.



- (a) Find the coordinates of the points where the graph of $y = f(x)$ intersects the coordinate axes. *(2 marks)*
- (b) (i) Find $\int (e^x - 3) dx$. *(2 marks)*
- (ii) Hence find the value of $\int_0^{\ln 3} (e^x - 3) dx$. *(2 marks)*
- (iii) Deduce that the area of the region shaded on the diagram is approximately 1.30. *(1 mark)*
- (c) (i) Find the range of the function f . *(1 mark)*
- (ii) Hence write down the domain and range of the inverse function f^{-1} , making it clear which is the domain and which is the range. *(2 marks)*
- (iii) Find an expression for $f^{-1}(x)$. *(3 marks)*
- (d) The function g is defined for all values of x by

$$g(x) = |x|.$$

- (i) Sketch the graph of $y = g(x)$. *(1 mark)*
- (ii) Copy the graph of $y = f(x)$ shown above, and on the same axes sketch the graph of

$$y = gf(x). \quad \text{span style="float: right;">*(2 marks)*$$

END OF QUESTIONS