General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A)   **MAS3**
Unit Statistics 3

Friday 25 June 2004   Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MAS3

Answer **all** questions.

1 Emma is learning to cook and wants to find a quick way of measuring out flour. Her father tells her that a heaped tablespoon of flour weighs 50 grams.

To investigate this statement, Emma spoons out one heaped tablespoon of flour and tips the flour onto a piece of paper. She carries out this process a total of nine times, tipping each tablespoon of flour onto a separate piece of paper. She then weighs each measure of flour separately and records its weight, x grams.

Emma's results are summarised below.

$$\text{Sample size} = 9 \qquad \sum x = 473 \qquad \sum x^2 = 24\,935$$

(a) Explain why Emma spooned out all the measures of flour and then weighed them, rather than weighing each measure as soon as she had spooned it out. *(1 mark)*

(b) Calculate unbiased estimates for the mean, μ, and the variance, σ^2, of the weight of a heaped tablespoon of flour spooned out by Emma. *(3 marks)*

(c) Stating the necessary distributional assumption, calculate a 90% confidence interval for:

  (i) the mean, μ; *(6 marks)*

  (ii) the standard deviation, σ. *(5 marks)*

(d) Use the confidence intervals you found in part (c) to comment on the usefulness, for Emma, of this method of measuring out flour. *(2 marks)*

2 The question master in a television quiz show asks exactly 80 questions in each show. Jamie watches the show regularly and always records the number of questions he can answer correctly as his score.

For a random sample of 10 shows, Jamie's scores are as follows.

$$58 \quad 48 \quad 40 \quad 38 \quad 54 \quad 51 \quad 34 \quad 63 \quad 39 \quad 59$$

(a) Jamie claims that, on average, his score is 50. Use a Wilcoxon signed-rank test, at the 5% significance level, to determine whether his claim is correct. *(7 marks)*

(b) Jamie later realises that he miscounted one of his scores. The last score in the above list should be 58 rather than 59.

  (i) State the changes that will occur in your list of signed ranks when this score is corrected. *(1 mark)*

  (ii) Hence explain why there will be no change to the conclusion you reached in part (a). *(1 mark)*

(c) Jamie's friend, Samir, also watches the show.

For a random sample of 15 shows, Jamie scores higher than Samir on 12 occasions and lower on 3 occasions.

Use an appropriate test and the 5% significance level to investigate whether, on average, Jamie scores higher than Samir. *(5 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ▶

3 The parents of children at a village primary school are concerned about the frequency of lorries passing the school. The Parents' Association organises a survey. Over a three-week period, volunteers stand outside the school, at randomly selected times during the school day, and record the time interval between consecutive lorries driving past in either direction.

When the survey is complete, the secretary of The Parents' Association summarises the data by drawing a histogram and calculating the mean and standard deviation of the time intervals between consecutive lorries. The results are shown below.

Distribution of Time Intervals between Consecutive Lorries

Sample mean = 3.49 minutes
Sample standard deviation = 3.66 minutes
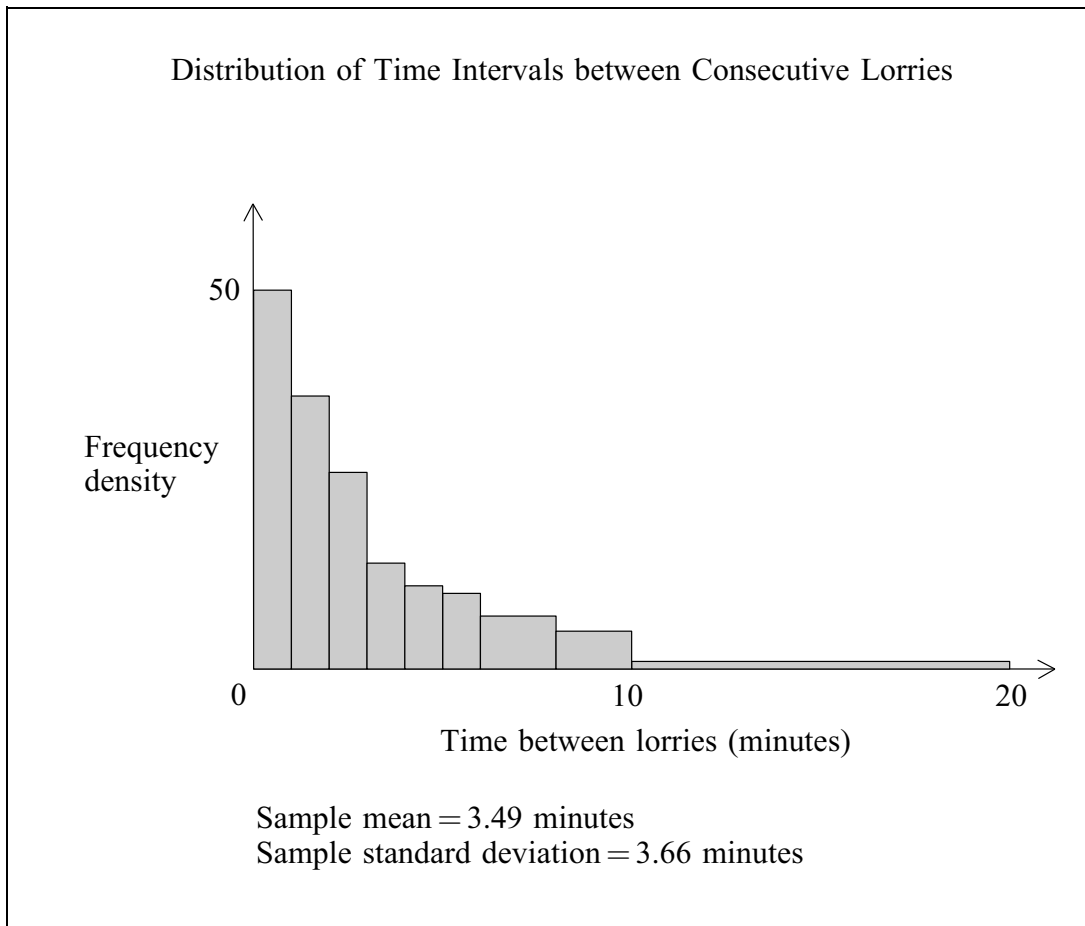
(a) Using the above summary of data, state reasons why it might be appropriate to use an exponential distribution to model the time intervals between consecutive lorries.

(2 marks)

(b) The distribution of the time interval, T minutes, between consecutive lorries is to be modelled as an exponential distribution with distribution function:

$$P(T \leqslant t) = F(t) = \begin{cases} 1 - e^{-0.3t} & t \geqslant 0 \\ 0 & t < 0. \end{cases}$$

(i) Find the mean value of T. *(1 mark)*

(ii) Find the probability that the time interval between consecutive lorries is less than one minute. *(2 marks)*

(iii) A school crossing patrol lady helps children to cross the road outside the school. One morning she steps out into the road to stop the traffic exactly one minute after a lorry has passed. She then holds up the traffic for exactly 45 seconds. Find the probability that no lorries are waiting when she allows the traffic to move. *(3 marks)*

(iv) Calculate the median time interval between consecutive lorries. *(3 marks)*

4 Jayne belongs to an athletics club and competes in the 200 metres. She believes that she performs better in competition than she does in practice.

The club keeps records of its members' running times for all practice runs and all competition runs. For Jayne's runs, the times, X seconds in practice and Y seconds in competition, are normally distributed variables.

Jayne takes random samples of 10 values of X and 7 values of Y. She then calculates unbiased estimates of the means, μ_X and μ_Y, and the variances, σ_X^2 and σ_Y^2, of the variables X and Y.

She obtains the following results.

	Unbiased estimate of mean	Unbiased estimate of variance
Practice runs	$\bar{x} = 28.76$	$s_X^2 = 1.60$
Competition runs	$\bar{y} = 27.60$	$s_Y^2 = 1.40$

(a) (i) Calculate a 90% confidence interval for the ratio, $\dfrac{\sigma_X^2}{\sigma_Y^2}$, of population variances. *(8 marks)*

(ii) Explain why it is reasonable to believe that $\sigma_X^2 = \sigma_Y^2$. *(1 mark)*

(b) Test, at the 5% significance level, Jayne's belief that she performs better in competition runs than in practice runs. *(9 marks)*

END OF QUESTIONS

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