GCE 2004

June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A

Unit MAP5

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Key to Mark Scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m marks and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
-*x* EE		deduct *x* marks for each error
NMS		no method shown
PI		possibly implied
SCA		substantially correct approach
c		candidate
SF		significant figure(s)
DP		decimal place(s)

Abbreviations used in Marking

MC – *x*	deducted *x* marks for mis-copy
MR – *x*	deducted *x* marks for mis-read
ISW	ignored subsequent working
BOD	given benefit of doubt
WR	work replaced by candidate
FB	formulae booklet

Application of Mark Scheme

No method shown:

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work	do not mark unless it has not been replaced
Alternative solution using a correct or partially correct method	award method and accuracy marks as appropriate

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MAP5

Q	Solution	Marks	Total	Comments
1(a)	$\dfrac{4}{x(x+4)} = \dfrac{1}{x} - \dfrac{1}{x+4}$	M1A1		Whole Q depends on the PFs
	$\mathrm{I} = \ln x - \ln(x+4)(+c)$	A1F	3	ft incorrect PFs
(b)(i)	$\mathrm{I} = \left[\ln x - \ln(x+4)\right]_0^1$	B1		attempt to put in limits
	$\ln x \to -\infty$ as $x \to 0 \therefore$ no finite limit	E1	2	
(ii)	$\dfrac{x}{x+4} \to 1$ as $x \to \infty$	E1		a clear explanation is required
	$\therefore \mathrm{I} = \ln 1 - \ln \dfrac{1}{5}$	M1		substitution of limits
	$= \ln 5$	A1F	3	O.E; no ln 1 in answer
	Total		**8**	
2	$\cos^k x = \left(1 - \dfrac{x^2}{2}\ldots\right)^k$	M1		
	$= 1 - \dfrac{kx^2}{2}\ldots$	A1		ignore higher powers of x
	$\lim\limits_{x \to 0} \dfrac{1 - \left(1 - \dfrac{kx^2}{2}\right)}{x^2} = 4$	M1		award only if some function of k appears
	$k = 8$	A1F	4	
	Total		**4**	

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MAP5 (Cont)

Q	Solution	Marks	Total	Comments
3(a)	$y_1 = 1 + h(1+1-3)$	M1		
	$= 1 - h$	A1	2	
(b)(i)	$x_1 = 1 + h$	B1		
	$y_2 = 1 + 2h\left((1+h)^2 + (1-h)^2 - 3\right)$	M1A1F		M0 if x_1 used throughout M1 if some function of h is used (including 1)
	$= 1 - 2h + 4h^3$	A1	4	AG
(ii)	$h = 0.05$	B1		B0 if $h = 0.1$
	$y(1.1) = y_2 = 1 - 2\times 0.05 + 4\times 0.05^3$ $= 0.9005$	B1F	2	Would have to accept to 3 sig fig ft $h = 0.1$ (giving 0.804)
	Total		**8**	
4	$2 = r + r\cos\theta$	M1		
	$= r + x$	B1		i.e. $x = r\cos\theta$ used relevantly
	$2 - x = r$	A1		
	$(2-x)^2 = x^2 + y^2$	M1		For relevant use of $r = \sqrt{x^2 + y^2}$
	$4 - 4x + x^2 = x^2 + y^2$	A1		
	$y^2 = 4(1-x)$	A1F	6	Or $y^2 = 4 - 4x$ o.e. ft simple arithmetical errors only
	Total		**6**	

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MAP5 (Cont)

Q	Solution	Marks	Total	Comments
5(a)	$\text{IF} = e^{-\int \frac{1}{x+1}dx} = e^{-\ln(x+1)}$	M1A1		
	$= \frac{1}{x+1}$	A1	3	
(b)	$\frac{d}{dx}\left(\frac{y}{x+1}\right) = \frac{x^2}{x+1}$	M1A1		
	$= \frac{1}{x+1} + x - 1$	M1A1F		
	$\frac{y}{x+1} = \frac{x^2}{2} - x + \ln(x+1) + c$	A1F		Allow if c missing
				Or by substituting $u = x+1$
				in this case $\int\left(u - 2 + \frac{1}{u}\right)du$ M1A1
	$c = 2$	A1F	6	$\frac{(x+1)^2}{2} - 2(x+1) + h(x+1) + c$ A1
	$y = (x+1)\left(\frac{x^2}{2} - x + \ln(x+1) + 2\right)$			$c = 3.5$ A1
(c)	$\lim_{x\to -1} y = 0$ since $(x+1)\ln(x+1) \to 0$ as $x \to -1$	E1	1	Must have proper explanation.
	Total		**10**	
6(a)	$R_1 + R_2 = \frac{1}{2}\int_{-(\pi-\alpha)}^{\alpha} 4(1-\cos\theta)^2 d\theta$	M1A1		M1 for use of formula A1 for correct limits (appearing at any point)
	$(1-\cos\theta)^2 = 1 - 2\cos\theta + \cos^2\theta$	A1		
	$\cos^2\theta = \frac{1+\cos 2\theta}{2}$ used	M1		
	$\text{I} = \left[3\theta - 4\sin\theta + \frac{\sin 2\theta}{2}\right]$	A1F		
	$a = 3,\ b = -8$	A1A1	7	CAO
(b)	$OA = 2\ (1-\cos\alpha)$	B1		
	$OB = 2(1-\cos(-\pi+\alpha))$	B1		Could use $\pi + \alpha$
	$AB = 4$	B1	3	
	Total		**10**	

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MAP5 (Cont)

Q	Solution	Marks	Total	Comments
7(a)	$\frac{du}{dx} = \frac{d^2y}{dx^2} - k\frac{dy}{dx}$	M1A1		
	$\frac{d^2y}{dx^2} - k\frac{dy}{dx} - k\left(\frac{dy}{dx} - ky\right) = 12xe^{kx}$	M1		M1 for everything in $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ or in $\frac{du}{dx}$, u and y
	$\frac{du}{dx} - ku = 12xe^{kx}$	A1	4	AG
(b)	IF is $e^{\int -k\,dx} = e^{-kx}$	B1		**Alternative method** CF $u = Ae^{kx}$ B1 PI $u = Bx^2e^{kx}$ M1 $\frac{du}{dx} = kBx^2e^{kx} + 2xBe^{kx}$ m1A1 $B = 6$ A1
	$\frac{d}{dx}\left(ue^{-kx}\right) = 12x$	M1A1		
	$ue^{-kx} = 6x^2 + A$	A1		A0 if A missing
	$u = \left(6x^2 + A\right)e^{kx}$	A1F	5	f.t. A missing
(c)	$\frac{dy}{dx} - ky = \left(6x^2 + A\right)e^{kx}$	M1		If attempt is made using *C.F.* and *P.I.* *C.F.* $y = (A + Bx)e^{kx}$ B1 *P.I.* $y = Cx^3e^{kx}$ M1 completely correct A1 total 3/5
	IF is e^{-kx}	B1		
	$\frac{d}{dx}\left(ye^{-kx}\right) = 6x^2 + A$	A1		
	$ye^{-kx} = 2x^3 + Ax + B$	A1		
	$y = (2x^3 + Ax + B)e^{kx}$	A1	5	6
	Total		**14**	
	Total		**60**	

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