GCE 2004
June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
Unit MAP4

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Key to Mark Scheme

M....................................mark is for .... method
m ....mark is dependent on one or more M marks and is for...................... method
A....................................mark is dependent on M or m marks and is for .... accuracy
B....................................mark is independent of M or m marks and is for ....method and accuracy
E....................................mark is for..explanation
√ or ft or F... follow through from previous incorrect result
CAO...correct answer only
AWFW ....anything which falls within
AWRT ....anything which rounds to
AG .... answer given
SC .... special case
OE.. or equivalent
A2,1... 2 or 1 (or 0) accuracy marks
-*x* EE...deduct *x* marks for each error
NMS...no method shown
PI ....possibly implied
SCA ....substantially correct approach
c.. candidate
SF...significant figure(s)
DP .... decimal place(s)

Abbreviations used in Marking

MC – *x*..deducted *x* marks for mis-copy
MR – *x*..deducted *x* marks for mis-read
ISW..ignored subsequent working
BOD..given benefit of doubt
WR...work replaced by candidate
FB ....formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working..mark as in scheme
Incorrect answer without working ....zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method award method and accuracy marks as appropriate

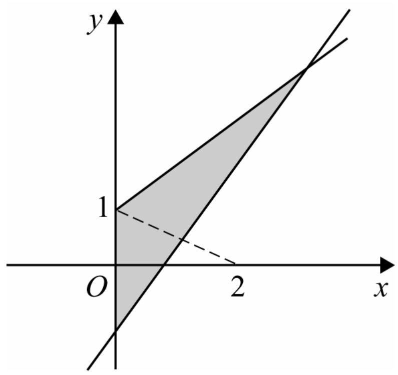
MAP4

Q	Solution	Marks	Total	Comments
1(a)	$(3-\mathrm{i})^2 = 9-6\mathrm{i}+\mathrm{i}^2 = 8-6\mathrm{i}$	B1	1	
(b)(i)	$a(8-6\mathrm{i})+b(3-\mathrm{i})+10\mathrm{i}=0$	M1		Substituting 3 – i into quadratic.
	Equating R & I parts	M1A1		
	$8a+3b=0$			
	$-6a-b+10=0$			
	Attempt to solve	M1		
	$a=3$, $b=-8$	A1A1F	6	$a=3$ is AG If $a = 3$ is assumed, allow M1A1 for b
(ii)	Sum of roots $=-\frac{b}{a}$	M1		If sum of roots is – 8 give M0
	or product = $\frac{c}{a}$			
	$\beta=-\frac{1}{3}+\mathrm{i}$	A1A1F	3	A1 for $-\frac{1}{3}$, A1 for + i
	Total		**10**	

MAP4 (Cont)

Q	Solution	Marks	Total	Comments
2(a)	$\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}=\frac{r+2-r}{r(r+1)(r+2)}$	M1		
	$=\frac{2}{r(r+1)(r+2)}$	A1	2	
(b)	$\frac{2}{1\times2\times3}=\frac{1}{1\times2}-\frac{1}{2\times3}$ $\frac{2}{2\times3\times4}=\frac{1}{2\times3}-\frac{1}{2\times4}$ $\frac{2}{3\times4\times5}=\frac{1}{3\times4}-\frac{1}{4\times5}$ $\frac{2}{30\times31\times32}=\frac{1}{30\times31}-\frac{1}{31\times32}$	M1A1		3 rows including first and last and clear cancellation for the A1 Accept last row in terms of n
	$S=\frac{1}{2}\left(\frac{1}{1\times2}-\frac{1}{31\times32}\right)$	M1A1		For substituting n = 30. Ignore missing $\frac{1}{2}$ for A1. Do not allow M1 if sum is left in terms of n.
	$=\frac{495}{1984}$	A1	5	cao
	Total		**7**	

MAP4 (Cont)

Q	Solution	Marks	Total	Comments
3(a)				
(i)	Straight line	B1		
	Perpendicular bisector of (0, 1) and (2, 0)	B1	2	Gradient must be > 1 i.e. greater than that of the other line.
(ii)	Half line	B1		
	through (0,1) with gradient ≈ 1	B1	2	
(b)	Correct identification of $\arg\ (z-\mathrm{i}) = -\dfrac{\pi}{2}$	B1		
	Shading on correct sides of boundaries	B2,1,0	3	For double shading or no shading at all without explanation, deduct B1
	Total		**7**	

MAP4 (Cont)

Q	Solution	Marks	Total	Comments
4(a)	$(y-3)^3+9(y-3)^2+27(y-3)+35=0$	M1A1		M1 for substituting
	$(y-3)^3=y^3-9y^2+27y-27$	M1A1		(a) Otherwise: $\sum(\alpha+3)=0$ M1A1; $\sum(\alpha+3)(\beta+3)=0$ M1A1; $(\alpha+3)(\beta+3)(\gamma+3)=-8$ M1A1
	$(y-3)^2=y^2-6y+9$	A1		
	$y^3+8=0$	A1	6	
(b)(i)	$y^3=-8e^{2k\pi i}$	M1		[Alternatie for 4(b)(i): $(a+ib)^3=-8$; $a^3-3ab^2=-8$, $3a^2b-b^3=0$ } M1; 2 values of a A1 $(-2,1)$ A1; 3 values of b A1 $(0\pm\sqrt{3})$ A1]
	$y=-2,\ -2e^{\frac{-2\pi i}{3}},\ -2e^{\frac{-2\pi i}{3}}$	A1		Or
	$=-2,\quad 1-\sqrt{3}i,\quad 1+\sqrt{3}i$	A1	3	[$(y^3+8=(y+2)(y^2-2y+4)$ M1; roots $-2,\ 1\pm\sqrt{3}i$ A2, 1, 0]
(ii)	α,β and γ are $-5,-2\pm\sqrt{3}i$	M1A1F	2	If 3 is added to the roots in (b)(i) allow M1A0
	Total		**11**	
5(a)	Attempt to expand $\left(z^2-\frac{1}{z^2}\right)^3$	M1		
	$A=-3,\quad B=1$	A1A1	3	
(b)(i)	$(2i\sin2\theta)^3=-3(2i\sin2\theta)+2i\sin6\theta$	M1A1F		Incorrect A, B
	$(2i\sin2\theta)^3=-8i\sin^3 2\theta$	A1F		
	$\sin^3 2\theta=\frac{3}{4}\sin2\theta-\frac{1}{4}\sin6\theta$	A1	4	AG
(ii)	$=\int_0^{\frac{1}{4}\pi}\sin^3 2\theta\,d\theta=\left[-\frac{3}{8}\cos2\theta+\frac{1}{24}\cos6\theta\right]_0^{\frac{1}{4}\pi}$	M1A1		If expression appears to be differentiated M0. Sign errors M1A0
	$=\frac{3}{8}-\frac{1}{24}=\frac{1}{3}$	A1	3	AG
	Total		**10**	

MAP4 (Cont)

Q	Solution	Marks	Total	Comments
6(a)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(2\tan^{-1}\mathrm{e}^{t}\right)=\frac{2}{1+\mathrm{e}^{2t}}\times\mathrm{e}^{t}$	M1A1 A1		$\frac{2}{1-\mathrm{e}^{2t}}$ M1A0
	$=\frac{2}{\mathrm{e}^{t}+\mathrm{e}^{-t}}$	M1		i.e. for dividing by e^{t} **Alternative** for last two marks
	$=\mathrm{sech}\,t$	A1	5	$\mathrm{sech}\,t=\frac{1}{\cosh t}=\frac{2}{\mathrm{e}^{t}+\mathrm{e}^{-t}}=\frac{2\mathrm{e}^{t}}{1+\mathrm{e}^{2t}}$ M1A1(2)
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}t}=(\cosh t)^{-2}\sinh t$	M1A1		$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{2\left(\mathrm{e}^{t}-\mathrm{e}^{-t}\right)}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}$ M1 only unless converted back into sech t and tanh t
	= secht tanht	A1	3	
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\mathrm{sech}^{2}t$	B1		P.I.
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^{2}+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^{2}=\mathrm{sech}^{4}t+\mathrm{sech}^{2}t\tanh^{2}t$			
	$=\mathrm{sech}^{2}t\left(\mathrm{sech}^{2}t+\tanh^{2}t\right)$	M1		Needs to be factorized for M1. M1 could be given for use of $\tanh^{2}t=1-\mathrm{sech}^{2}t$
	$=\mathrm{sech}^{2}t$	A1	3	CAO
(c)(i)	$S=2\pi\int_{t=0}^{t=1}y\,\mathrm{d}s$			
	$=2\pi\int_{0}^{1}(2-\mathrm{sech}\,t)\mathrm{sech}\,t\,\mathrm{d}t$	B1	1	AG must be from correct (b)(ii) i.e. correct working
(ii)	$=2\pi\left[4\tan^{-1}\mathrm{e}^{t}-\tanh t\right]_{0}^{1}$	B1B1		
	$=2\pi\left[4\tan^{-1}\mathrm{e}-\tanh 1-\pi\right]$	B1	3	AG
	Total		**15**	
	Total		**60**	