

General Certificate of Education  
June 2004  
Advanced Level Examination



**MATHEMATICS (SPECIFICATION A)**  
**Unit Pure 3**

**MAP3**

Wednesday 23 June 2004 Afternoon Session

**In addition to this paper you will require:**

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

**Information**

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

**Advice**

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

---

Answer **all** questions.

---

1 A curve is given by the parametric equations

$$x = 2t - 1, \quad y = \frac{1}{2t}.$$

(a) Find  $\frac{dy}{dx}$  in terms of  $t$ . (2 marks)

(b) Find the equation of the normal to the curve at the point where  $t = 1$ . (4 marks)

2 (a) Obtain the binomial expansion of  $(1 + x)^{\frac{1}{3}}$  as far as the term in  $x^2$ . (2 marks)

(b) Hence, or otherwise, find the series expansion of  $(8 + 4x)^{\frac{1}{3}}$  as far as the term in  $x^2$ .  
(3 marks)

3 (a) Express  $\frac{30}{(x + 4)(7 - 2x)}$  in the form  $\frac{A}{x + 4} + \frac{B}{7 - 2x}$ . (3 marks)

(b) Hence find

$$\int_0^3 \frac{30}{(x + 4)(7 - 2x)} dx,$$

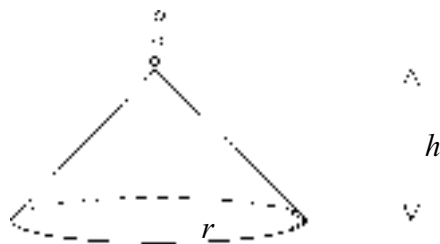
giving your answer in the form  $p \ln q$ , where  $p$  and  $q$  are rational numbers. (5 marks)

4 A curve is given by the equation  $9(y + 2)^2 = 5 + 4(x - 1)^2$ .

(a) Find the coordinates of the two points on the curve where  $x = 2$ . (3 marks)

(b) Find the gradient of the curve at each of these points. (5 marks)

- 5 In a timing device, sand falls through a small hole to form a conical heap. As the cone forms, the height,  $h$  cm, remains equal to the base radius,  $r$  cm, of the heap.



- (a) The volume of the sand after  $t$  minutes is  $V$  cm<sup>3</sup>. Explain why  $V = \frac{1}{3}\pi h^3$ . (1 mark)
- (b) The sand falls through the hole at a rate of 3 cm<sup>3</sup> per minute. Find the rate at which the height of the heap is increasing at the instant when  $h = 2$ . Give your answer to two significant figures. (3 marks)
- 6 The function  $f$  is given by  $f(x) = \sin\left(2x + \frac{\pi}{6}\right)$ .

- (a) (i) Find  $f'(x)$  and  $f''(x)$ . (3 marks)
- (ii) Hence, using the Maclaurin series, show that, for small values of  $x$ ,

$$f(x) \approx \frac{1}{2} + \sqrt{3}x - x^2. \quad (2 \text{ marks})$$

- (b) Show that, for small values of  $x$ ,

$$(1 - \cos x) \sin\left(2x + \frac{\pi}{6}\right) \approx kx^2,$$

where the value of the constant  $k$  is to be found. (3 marks)

**TURN OVER FOR THE NEXT QUESTION**

Turn over ►

- 7 Initially there are 2000 fish in a lake. The number of fish,  $x$ , at time  $t$  months later is modelled by the differential equation

$$\frac{dx}{dt} = x(1 - kt),$$

where  $k$  is a constant.

- (a) Solve this differential equation to show that

$$x = 2000e^{t - \frac{1}{2}kt^2}. \quad (6 \text{ marks})$$

- (b) After 12 months the number of fish is again 2000. Find the value of  $k$ . (3 marks)

- 8 (a) Find the vector equation of the line  $l_1$ , which passes through the points  $A(3, -1, 2)$  and  $B(2, 0, 2)$ . (2 marks)

(b) The line  $l_2$  has vector equation  $\mathbf{r} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ .

Show that the lines  $l_1$  and  $l_2$  intersect and find the coordinates of their point of intersection. (4 marks)

- (c) Show that the point  $C(9, 1, -6)$  lies on the line  $l_2$ . (2 marks)

- (d) Find the coordinates of the point  $D$  on  $l_1$  such that  $CD$  is perpendicular to  $l_1$ . (4 marks)

**END OF QUESTIONS**