

General Certificate of Education
June 2004
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Pure 2

MAP2

Wednesday 16 June 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- an insert for use in Question 5 (enclosed);
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used, including the insert for use in Question 5, to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 (a) Find the coordinates of the points where the graph of $y = \frac{2x+1}{x-1}$ crosses:
- (i) the y -axis; *(1 mark)*
 - (ii) the x -axis. *(1 mark)*
- (b) (i) Express $\frac{2x+1}{x-1}$ in the form $A + \frac{B}{x-1}$. *(3 marks)*
- (ii) Write down the equations of the two asymptotes to the graph of $y = \frac{2x+1}{x-1}$. *(2 marks)*
- (c) Hence sketch the graph of $y = \frac{2x+1}{x-1}$. *(3 marks)*
- (d) By using your graph, or otherwise, solve the inequality

$$\frac{2x+1}{x-1} \leq 0. \quad \text{(2 marks)}$$

- 2 (a) Show that $\sin(\alpha + \beta) + \sin(\alpha - \beta) \equiv 2 \sin \alpha \cos \beta$. *(2 marks)*
- (b) (i) Express $2 \sin 8x \cos 2x$ in the form $\sin A + \sin B$. *(2 marks)*
- (ii) Hence find $\int 6 \sin 8x \cos 2x \, dx$. *(3 marks)*

- 3 (a) Use integration by parts to evaluate $\int_0^{\frac{\pi}{2}} x \cos x \, dx$. *(5 marks)*
- (b) (i) Use the substitution $t = x^2 + 4$ to show that $\int \frac{2x \, dx}{\sqrt{x^2 + 4}} = \int \frac{1}{\sqrt{t}} \, dt$. *(2 marks)*
- (ii) Show that $\int_0^2 \frac{2x \, dx}{\sqrt{x^2 + 4}} = 4(\sqrt{2} - 1)$. *(4 marks)*

4 (a) (i) Find $\frac{dy}{dx}$ when $y = e^x \sin 2x$. (3 marks)

(ii) Hence find the equation of the tangent to the curve $y = e^x \sin 2x$ at the origin. (2 marks)

(b) Show that the equation of the normal to the curve $y = e^x \sin 2x$ at the point where $x = \pi$ is

$$2e^\pi y + x = \pi. \quad (4 \text{ marks})$$

5 [Figure 1, printed on the insert, is provided for use in answering this question.]

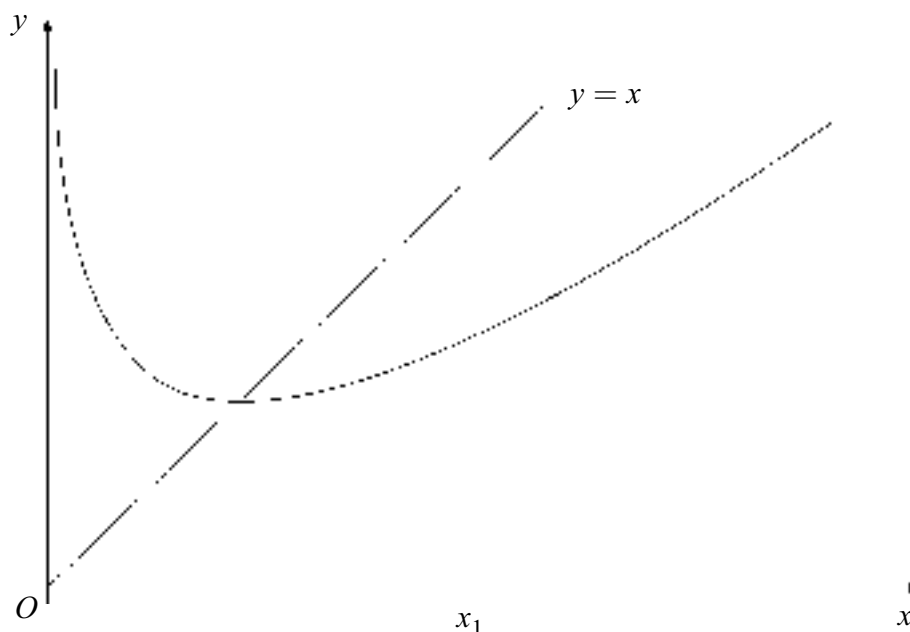
(a) Show, without using a calculator, that the equation $x^3 - 15 = 0$ has a root in the interval $2 \leq x \leq 3$. (2 marks)

(b) (i) Show that the equation $x = \frac{2x}{3} + \frac{5}{x^2}$ can be rearranged to give the equation

$$x^3 - 15 = 0. \quad (2 \text{ marks})$$

(ii) Use the iterative formula $x_{n+1} = \frac{2x_n}{3} + \frac{5}{x_n^2}$, starting with $x_1 = 3$, to find the values of x_2 , x_3 and x_4 , giving your answers to six decimal places. (4 marks)

(iii) The graphs of $y = \frac{2x}{3} + \frac{5}{x^2}$ and $y = x$ are sketched below.



On Figure 1, draw a staircase diagram to illustrate the convergence of the sequence x_1, x_2, x_3, \dots (2 marks)

(iv) Write down the **exact** value to which this sequence converges. (1 mark)

Turn over ►

- 6 (a) The circle $(x - 4)^2 + (y - 3)^2 = 4$ has centre C and radius r .

Write down:

- (i) the coordinates of C ;
 - (ii) the value of r . *(2 marks)*
- (b) The line $y = x + 1$ intersects this circle at two points A and B .
- (i) Find the coordinates of A and B . *(5 marks)*
 - (ii) Show that the area of the minor segment bounded by the circle and the chord AB is $\pi - 2$. *(3 marks)*

END OF QUESTIONS

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|---------------------|--|--|--|--|--|------------------|--|--|--|--|--|
| Surname | | | | | | Other Names | | | | | |
| Centre Number | | | | | | Candidate Number | | | | | |
| Candidate Signature | | | | | | | | | | | |

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Insert for use in answering Question 5.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over ►

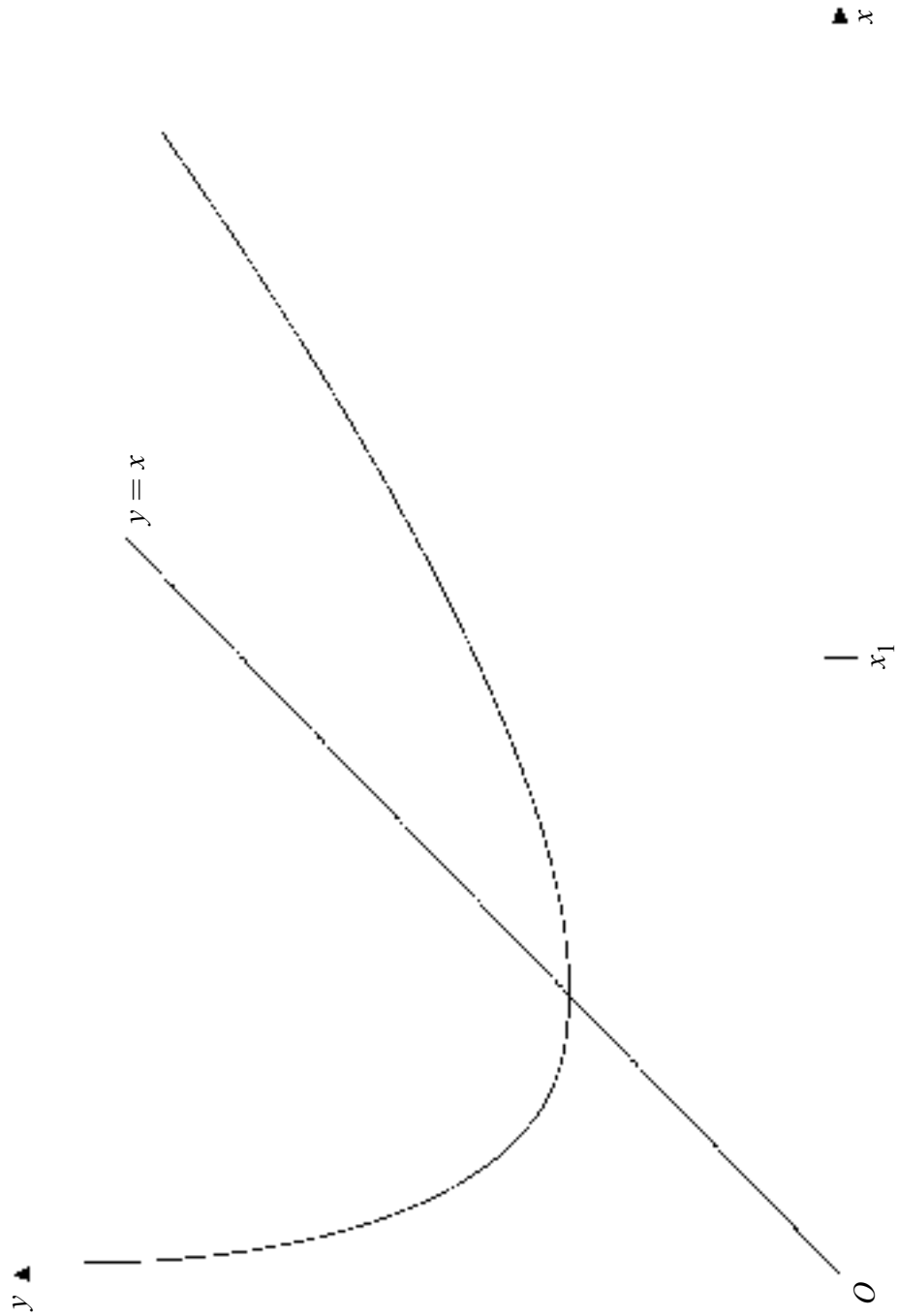


Figure 1 (for Question 5)