General Certificate of Education
June 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAM4/W**
Unit Mechanics 4

Tuesday 29 June 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM4/W.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A small unmanned submarine of mass 800 kg moves in a straight horizontal line under the action of a propulsive force of $20\,\mathrm{e}^{t}$ N, $0 \leqslant t \leqslant 6$, and a resistive force of magnitude $400v$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of the submarine at time t seconds.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{1}{2}v = \frac{1}{40}\mathrm{e}^{t}.$$

(3 marks)

(b) Using an integrating factor, or otherwise, find the general solution of this differential equation. *(5 marks)*

(c) Given that the submarine is at rest when $t = 0$, find the speed of the submarine when $t = 6$. *(3 marks)*

2 A rocket is launched vertically upwards from the surface of the Earth. The initial total mass of the rocket and its fuel is M. The rocket ejects a mass of burnt fuel at a constant rate α and with a constant exhaust speed u relative to the rocket. At time t, the rocket has mass m and is travelling with speed v.

(a) By using the impulse/momentum principle, show that the equation of motion of the rocket is

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{u\alpha}{M - \alpha t} - g\,.$$

(7 marks)

(b) The initial mass of the fuel is $\frac{2}{3}M$.

Show that when $t = \dfrac{2M}{3\alpha}$, the rocket's motor will stop running. *(1 mark)*

(c) By using the results in parts (a) and (b) and assuming that the rocket starts to rise when $t = 0$, find, in terms of u, M, g and α, the velocity of the rocket at the instant when the motor stops running. *(6 marks)*

3 A merry-go-round has a horizontal circular platform of radius 4 m. The platform is rotating about a vertical axis through its centre at a constant rate of one revolution every 5 seconds.

Jason is of mass 60 kg and he walks along a radius of the platform from the centre to the edge. Assume that Jason walks along the radius at a constant speed of 2 m s^{-1} without any obstructions and he is not thrown off balance.

(a) Find the angular velocity of the merry-go-round in radians per second. *(1 mark)*

(b) Find the magnitude of the transverse force which Jason experiences. *(2 marks)*

(c) Determine the magnitude of the maximum central force exerted on Jason. *(2 marks)*

4 On an island there are rabbits and foxes. The rate of growth of the number of foxes on the island is known to be proportional to the number of rabbits. The rate of change of population of rabbits is dependent on both the number of rabbits and the number of foxes. The relationships between the two populations are modelled by the differential equations

$$12\dot{x} = y \qquad \text{and} \qquad \dot{y} = 2y - 9x\,,$$

where x and y are the numbers of foxes and rabbits respectively after time t years.

(a) By eliminating x between the two equations, show that y satisfies the differential equation

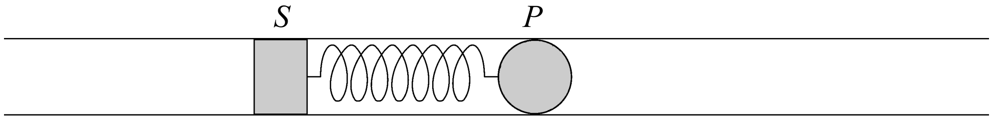
$$4\ddot{y} - 8\dot{y} + 3y = 0\,.$$ *(3 marks)*

(b) Initially the number of foxes is 500 and the number of rabbits is n. Find y in terms of n and t. *(8 marks)*

(c) Given that $n = 2800$, find the time when the population of rabbits dies away. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

5 A solid ball P is attached to one end of a light elastic spring. The other end of the spring is attached to a piston S which is free to move horizontally inside a smooth tube, as shown in the diagram.



The mass of P is 0.5 kg. The spring has natural length 0.5 m and stiffness $4.5\ \text{N m}^{-1}$. The system initially rests in equilibrium.

The piston S is forced to oscillate, causing P to move. At time t seconds, the displacements of S and P are $\frac{1}{2}\sin 2t$ metres and x metres respectively, measured in the same direction from their initial positions.

(a) Draw a diagram showing the forces acting on P and the displacements of P and S. *(1 mark)*

(b) Show that x satisfies the differential equation

$$\ddot{x} + 9x = \frac{9}{2}\sin 2t\,.$$ *(4 marks)*

(c) Find x as a function of t. *(9 marks)*

(d) Show that P is instantaneously at rest when $t = \frac{2\pi}{5}$. *(2 marks)*

END OF QUESTIONS