GCE 2004
June Series

ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark Scheme

Mathematics A
Unit MAM4/W

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Key to Mark Scheme

M ..... mark is for ..... method
m ..... mark is dependent on one or more M marks and is for ..... method
A ..... mark is dependent on M or m marks and is for ..... accuracy
B ..... mark is independent of M or m marks and is for ..... method and accuracy
E ..... mark is for ..... explanation
√ **or ft or F** ..... follow through from previous incorrect result
CAO ..... correct answer only
AWFW ..... anything which falls within
AWRT ..... anything which rounds to
AG ..... answer given
SC ..... special case
OE ..... or equivalent
A2,1 ..... 2 or 1 (or 0) accuracy marks
-*x* EE ..... deduct *x* marks for each error
NMS ..... no method shown
PI ..... possibly implied
SCA ..... substantially correct approach
c ..... candidate
SF ..... significant figure(s)
DP ..... decimal place(s)

Abbreviations used in Marking

MC – *x* ..... deducted *x* marks for mis-copy
MR – *x* ..... deducted *x* marks for mis-read
ISW ..... ignored subsequent working
BOD ..... given benefit of doubt
WR ..... work replaced by candidate
FB ..... formulae booklet

Application of Mark Scheme

No method shown:
Correct answer without working ..... mark as in scheme
Incorrect answer without working ..... zero marks unless specified otherwise

More than one method/choice of solution:

2 or more complete attempts, neither/none crossed out	mark both/all fully and award the mean mark rounded down
1 complete and 1 partial attempt, neither crossed out	award credit for the complete solution only

Crossed out work — do not mark unless it has not been replaced

Alternative solution using a correct or partially correct method — award method and accuracy marks as appropriate

MAM4/W

Q	Solutions	Marks	Total	Comments
1(a)	$20e^{t} - 400v = 800\dfrac{dv}{dt}$	M1 A1		Newton's 2nd law
	$\dfrac{dv}{dt} + \dfrac{1}{2}v = \dfrac{1}{40}e^{t}$	A1	3	AG
(b)	Integrating factor is $e^{\frac{1}{2}t}$	M1 A1		
	$e^{\frac{1}{2}t}\dfrac{dv}{dt} + \dfrac{1}{2}e^{\frac{1}{2}t}v = \dfrac{1}{40}e^{\frac{3}{2}t}$			
	$e^{\frac{1}{2}t}v = \int \dfrac{1}{40}e^{\frac{3}{2}t}dt$	M1 A1		Correct form
	$e^{\frac{1}{2}t}v = \dfrac{1}{60}e^{\frac{3}{2}t} + C$	A1F	5	
(c)	$v = 0, t = 0 \Rightarrow C = -\dfrac{1}{60}$	M1		
	$e^{3}v = \dfrac{1}{60}e^{9} - \dfrac{1}{60}$	M1		Substituting $t = 6$
	$v = 6.72$	A1F	3	At least 3 s.f. required for A1
	Total		**11**	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
2(a)	m $\uparrow v$ Time t m-δm $\uparrow v+\delta v$ δm $\downarrow u-v$ Time $t+\delta t$	B1		p.i.
	$(m-\delta m)(v+\delta v)+\delta m(v-u)-mv=-mg\delta t$	M1 A2,1		Attempt to use the principle –1 ee
	$m\frac{\delta v}{\delta t}-\frac{\delta m}{\delta t}\delta v-u\frac{\delta m}{\delta t}=-mg$	m1		First order
	$m\frac{\mathrm{d}v}{\mathrm{d}t}-u\frac{\mathrm{d}m}{\mathrm{d}t}=-mg$			
	$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{u}{m}\frac{\mathrm{d}m}{\mathrm{d}t}-g$	m1		
	$\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{u\alpha}{M-\alpha t}-g$	A1	7	AG
(b)	The motor will stop when all fuel is burnt.			
	$\frac{2}{3}M=\alpha t$	B1	1	o.e.
	$t=\frac{2M}{3\alpha}$			AG
(c)	$\int_0^v \mathrm{d}v=\int_0^t(\frac{u\alpha}{M-\alpha t}-g)\mathrm{d}t$			
	$v=-u\ln(M-\alpha t)-gt+u\ln M$	M1		–1 ee
	$v=-u\ln(M-\alpha\frac{2M}{3\alpha})-g\frac{2M}{3\alpha}+u\ln M$	A2,1 m1 A1F		
	$v=-u\ln\frac{1}{3}M+u\ln M-\frac{2gM}{3\alpha}$			
	$v=u\ln 3-\frac{2gM}{3\alpha}$	A1F	6	
	Total		**14**	

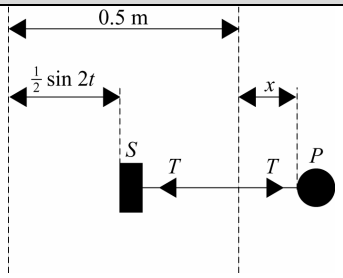
MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
3(a)	$\dot{\vartheta} = \frac{2\pi}{5}$ rad s^{-1}	B1	1	Or equivalent
(b)	Transverse force $F = m(r\ddot{\vartheta} + 2\dot{r}\dot{\vartheta})$			
	$= 60(2 \times 2 \times \frac{2\pi}{5})$	M1		
	$= 96\,\pi$ N (= 302 N)	A1	2	
(c)	Max. Central Force $F = -m(\ddot{r} - r\dot{\vartheta}^2)$			
	$= -60(-4(\frac{2\pi}{5})^2)$	M1		
	$= \frac{192}{5}\pi^2$ N (=379N)	A1	2	
	Total		**5**	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
4(a)	$12\dot{x} = y$			
	$\dot{y} = 2y - 9x$			
	$\ddot{y} = 2\dot{y} - 9\dot{x}$	M1		
	$\ddot{y} - 2\dot{y} + 9(\frac{y}{12}) = 0$	ml		Substitution for $\dot{x}$
	$4\ddot{y} - 8\dot{y} + 3y = 0$	A1	3	
(b)	$4m^2 - 8m + 3 = 0$	M1		Attempt at solving aux
	$m = \frac{1}{2}, \frac{3}{2}$	A1		
	$y = Ae^{\frac{1}{2}t} + Be^{\frac{3}{2}t}$	M1		
	$y = n, x = 500, t = 0$			
	$n = A + B$	ml		
	$\dot{y} = \frac{1}{2}Ae^{\frac{1}{2}t} + \frac{3}{2}Be^{\frac{3}{2}t}$			
	$2y - 9x = \frac{1}{2}Ae^{\frac{1}{2}t} + \frac{3}{2}Be^{\frac{3}{2}t}$	ml		
	$2n - 4500 = \frac{1}{2}A + \frac{3}{2}B$	ml		
	$B = \frac{3}{2}n - 4500, A = 4500 - \frac{1}{2}n$	A1F A1F	8	
(c)	$n = 2800$			
	$y = 3100e^{\frac{1}{2}t} - 300e^{\frac{3}{2}t}$			
	$y = 0 \Rightarrow e^t = \frac{3100}{300}$	M1		For setting $y = 0$ & $n = 2800$
	$t = \ln\frac{3100}{300}$	ml		
	$t = 2.34$	A1F	3	AWRT
	Total		**14**	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
5(a)		B1	1	
(b)	$T = 4.5(\frac{1}{2}\sin 2t - x)$	M1 A1		
	$T = 0.5\ddot{x}$			
	$0.5\ddot{x} = 4.5(\frac{1}{2}\sin 2t - x)$	M1		Newton's 2nd law
	$\ddot{x} + 9x = \frac{9}{2}\sin 2t$	A1	4	AG
(c)	Auxiliary eq^{n}. $m^2 + 9 = 0$	M1		
	$m = \pm 3\text{i}$	A1		
	C.F. $x = A\text{Cos}3t + B\text{Sin}3t$	B1		
	For P.I. $x = p\cos 2t + q\sin 2t$			
	$\dot{x} = -2p\sin 2t + 2q\cos 2t$			
	$\ddot{x} = -4p\cos 2t - 4q\sin 2t$	m1		
	$5p\cos 2t + 5q\sin 2t = \frac{9}{2}\sin 2t$	M1		
	$p = 0, q = \frac{9}{10}$	A1F		For both p and q
	G.S. is			
	$x = A\cos 3t + B\sin 3t + \frac{9}{10}\sin 2t$	A1F		
	$x = \dot{x} = 0, t = 0$			
	$\therefore A = 0$	A1F		
	$\dot{x} = 3B\cos 3t + \frac{9}{5}\cos 2t$			
	$\therefore B = -\frac{3}{5}$	A1F		
	$x = -\frac{3}{5}\sin 3t + \frac{9}{10}\sin 2t$		9	

MAM4/W (Cont)

Q	Solution	Marks	Total	Comments
5 (d)	$\dot{x} = -\frac{9}{5}\cos 3t + \frac{9}{5}\cos 2t$ $= -\frac{9}{5}\cos\frac{6\pi}{5} + \frac{9}{5}\cos\frac{4\pi}{5}$ $= 0$	M1 A1	 2	
	Total		**16**	
	Total		**60**	