

General Certificate of Education
June 2004
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Mechanics 3

MAM3

Friday 25 June 2004 Morning Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM3.
- Answer **all** questions.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

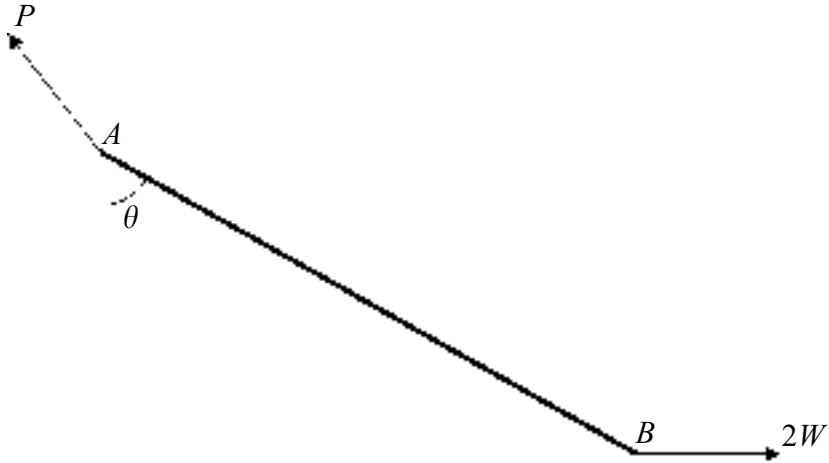
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 The diagram shows a uniform rod AB of length $2a$ and weight W . The end A is freely hinged to a fixed support. A horizontal force $2W$ is applied at B to keep the rod in equilibrium as shown in the diagram below.



- (a) Write down the horizontal and vertical components of the reaction, P , of the hinge on the rod at A . (2 marks)
- (b) Find the value of the angle θ which the rod makes with the downward vertical. (3 marks)
- (c) With the aid of a diagram, describe the position of one point, other than A , through which the line of action of P must pass. (2 marks)

- 2 A high diver uses a special harness in a gymnasium to enable him to practise somersaults. The harness consists of a strong belt about his waist. It is attached by smoothly swivelling connections to inextensible supporting cables. The harness keeps the diver's waist at the same horizontal level throughout his somersaults.

Assume that the diver's centre of mass remains at his waist level throughout the motion.

- (a) **Figure 1** shows the diver's body in a tight 'tucked' or curled up position.

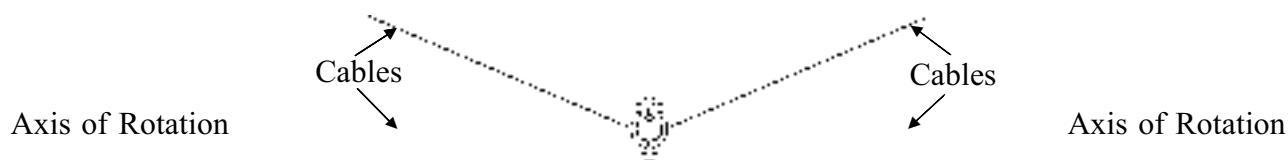


Figure 1

In this position he completes 2 somersaults per second at a constant angular speed. Each somersault consists of a complete rotation of his body. His moment of inertia in this position is 1.5 kg m^2 .

Calculate his angular momentum.

(3 marks)

- (b) He continues rotating and smoothly changes to a stretched out position as shown in **Figure 2**.

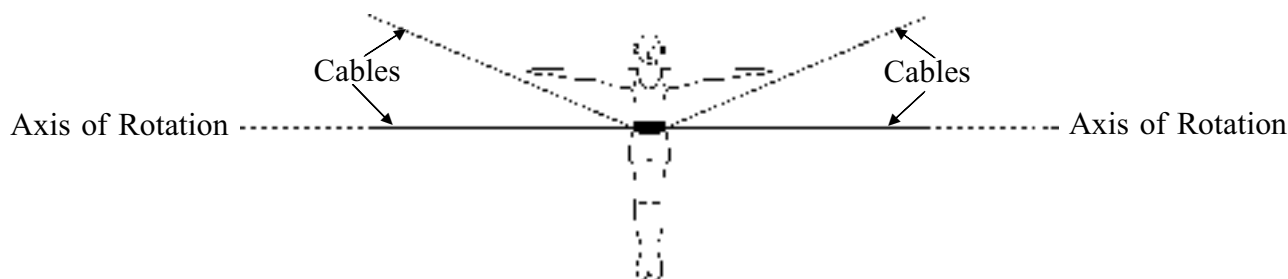


Figure 2

His moment of inertia in this new position is 8 kg m^2 .

Calculate his new angular speed.

(2 marks)

TURN OVER FOR THE NEXT QUESTION

Turn over ►

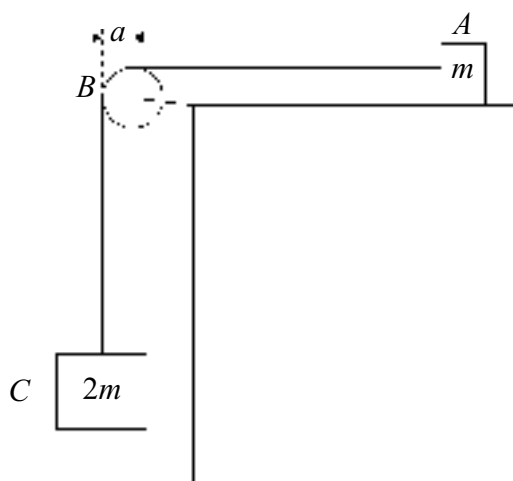
3 Forces $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ act in the x - y plane at the points with coordinates $(0, 0)$, $(0, 2)$, $(3, 4)$ and $(3, 0)$ respectively.

- (a) Find the magnitude of the resultant \mathbf{F} of this system of forces. (4 marks)
- (b) The line of action of \mathbf{F} cuts the x -axis at the point $(d, 0)$. By taking moments about the origin, or otherwise, find d . (4 marks)
- (c) The system of forces is equivalent to a force \mathbf{F} acting at the origin, together with a couple L . Write down the magnitude of L and indicate its sense on a diagram. (2 marks)

4 The diagram shows a particle A initially held at rest on a smooth, horizontal table.

A pulley B is fixed to the edge of the table. The pulley is free to rotate in a vertical plane about a horizontal axis through its centre and can be modelled as a uniform disc of mass $10m$ and radius a .

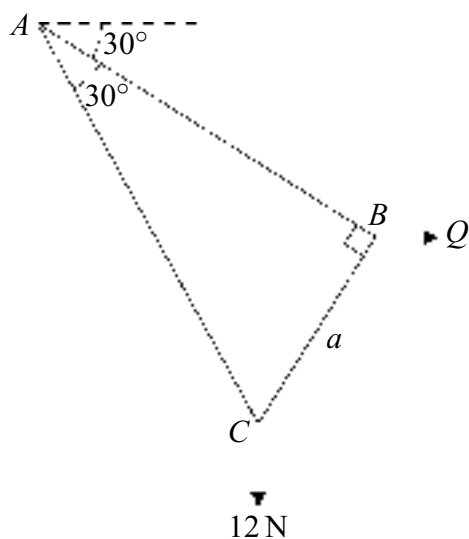
A light inextensible string runs over the pulley, connecting the particle A , of mass m , to a particle C , of mass $2m$, which is hanging freely.



- (a) Find the moment of inertia of the pulley about the horizontal axis through its centre and perpendicular to its plane. (1 mark)
- (b) Particle A is released. In the subsequent motion, the section of string between A and B remains horizontal, and the section of string between B and C remains vertical. The string does **not** slip when in contact with the pulley.

Show that, whilst A moves on the table, the angular acceleration of the pulley is $\frac{g}{4a}$. (8 marks)

- 5 The diagram shows a framework of three light rods AB , BC and CA in the same vertical plane smoothly pin-jointed at A , B and C . The framework is freely hinged to a smooth support at A . Angle $BAC = 30^\circ$ and angle $ABC = 90^\circ$. The rod BC is of length a . A weight of 12 newtons hangs from C and a horizontal force Q is applied to the point B so that AB makes an angle of 30° with the horizontal.



- (a) By considering the forces on the whole framework, find the value of the force Q .
(3 marks)
- (b) (i) Find the magnitudes of the forces in AB and BC .
(6 marks)
- (ii) State whether AB is in compression or tension.
(1 mark)
- (iii) State whether BC is in compression or tension.
(1 mark)
- (c) Find the magnitude of the force in AC .
(2 marks)

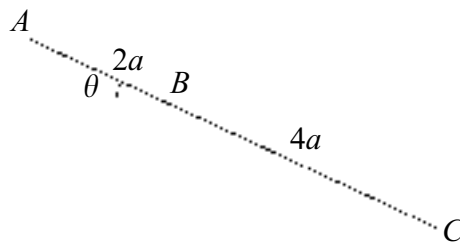
TURN OVER FOR THE NEXT QUESTION

Turn over ►

- 6 A uniform straight rod ABC , of mass m , has length $6a$, where $AB = 2a$ and $BC = 4a$.



- (a) Show that the moment of inertia of the rod about an axis through the point B and perpendicular to the rod is $4ma^2$. (2 marks)
- (b) The rod is held at rest on a horizontal table perpendicular to the edge so that a length $4a$ projects horizontally over the edge. The rod is released from rest and begins to rotate about the point B and in a vertical plane. The rod remains in contact with the edge of the table.



When the rod has turned through an angle θ , and has not yet begun to slip:

- (i) use the principle of conservation of energy to show that the angular speed of the rod is $\sqrt{\frac{g \sin \theta}{2a}}$; (3 marks)
- (ii) find an expression, in terms of a , g and θ , for the angular acceleration of the rod; (3 marks)
- (iii) determine the component perpendicular to the rod of the reaction force acting on the rod at B . (3 marks)
- (c) Show that the rod begins to slip when $\tan \theta = \frac{\mu}{2}$, where μ is the coefficient of friction between the table and the rod. (5 marks)

END OF QUESTIONS

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