

General Certificate of Education
January 2004
Advanced Level Examination



MATHEMATICS (SPECIFICATION A)
Unit Statistics 3

MAS3

Tuesday 27 January 2004 Afternoon Session

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAS3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information

- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 A machine produces steel bolts of a particular diameter. A quality control manager finds that more bolts than usual are too large and more bolts than usual are too small. She decides to investigate whether there has been an increase in the standard deviation of the diameters of bolts produced.

The manager takes a random sample of 10 bolts and measures the diameter, x millimetres, of each bolt. She calculates the following totals.

$$\sum x = 75.43 \quad \sum x^2 = 569.45$$

- (a) Calculate an unbiased estimate of the variance of the diameters of bolts produced by the machine. *(2 marks)*
- (b) The manager knows from previous experience that the standard deviation of diameters should be 0.15 millimetres. Stating the necessary distributional assumption, test, at the 5% significance level, her suspicion that the standard deviation is higher than it should be. *(7 marks)*
- (c) Explain why the manager decided to test for an increase in the standard deviation rather than for a change in the mean. *(2 marks)*
- 2 Kiran travels to and from school by bike. His journey times in both directions are normally distributed. Kiran randomly selects two samples, each of 12 dates during the autumn term. On each date in the first sample, he records his journey time, x minutes, from home to school. On each date in the second sample, he records his journey time, y minutes, from school to home.

Kiran uses these data to calculate an unbiased estimate, s_X^2 , of the variance of journey times from home to school and an unbiased estimate, s_Y^2 , of the variance of journey times from school to home. His results are as follows.

$$s_X^2 = 2.4049 \quad s_Y^2 = 0.5372$$

- (a) Calculate a 95% confidence interval for the ratio, $\frac{\sigma_X^2}{\sigma_Y^2}$, of population variances for Kiran's journeys to and from school. *(7 marks)*
- (b) Use your confidence interval to compare the variability in Kiran's journey times to and from school. *(2 marks)*

- 3 A coffee producer sells two types of instant coffee granules: an economy blend and a luxury blend. Bethany carries out an experiment to investigate whether coffee made from granules of the luxury blend tastes better than coffee made from granules of the economy blend.

Bethany asks a random sample of 12 people to taste both types of coffee and to give each type a score for taste on a scale of 1 (poor) to 10 (excellent). She presents the two blends to the tasters in random order so that they do not know which blend they are tasting.

The results are given in the table below.

Taster	A	B	C	D	E	F	G	H	I	J	K	L
Score for economy blend	4	6	5	2	6	7	8	3	5	2	8	6
Score for luxury blend	7	5	9	10	6	8	7	10	5	5	8	8

- (a) Carry out a sign test, at the 10% significance level, to determine whether there is a taste preference for coffee made from granules of the luxury blend. *(7 marks)*
- (b) (i) Explain briefly why it is preferable, if possible, to use a Wilcoxon signed-rank test rather than a sign test. *(1 mark)*
- (ii) Give a reason why it would **not** be appropriate to use a Wilcoxon signed-rank test for the above investigation. *(1 mark)*

TURN OVER FOR THE NEXT QUESTION

Turn over ►

- 4 The pupils in a school are affected by an outbreak of flu. Any pupil who catches flu is ill for at least three full days. After this three-day period there is a **further** time, T days, until the pupil has fully recovered.

The random variable, T , is exponentially distributed with the following probability density function.

$$f(t) = \begin{cases} \frac{1}{2}e^{-\frac{t}{2}} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- (a) Show that, for $t \geq 0$, T has distribution function

$$F(t) = 1 - e^{-\frac{t}{2}}. \quad (2 \text{ marks})$$

- (b) (i) Janice has been ill with flu for exactly three days. Calculate the probability that she will have fully recovered in a **further** three days. (2 marks)
- (ii) Kevin has just caught flu. Calculate the probability that he will **not** have fully recovered in four days' time. (3 marks)
- (iii) Lucy has been ill with flu for exactly four days. Calculate the probability that she is ill with flu for a **total** of less than six days. (4 marks)
- (c) The random variable, X , denotes the **total time**, in days, for which a pupil is ill with flu.

Write down values for the mean and standard deviation of X . (3 marks)

- 5 A woodland area in France is home to a population of dormice. The weights of adult male dormice in this population are normally distributed.

A conservationist takes a random sample of 8 adult male dormice in June and records the weight, x grams, of each dormouse. He calculates the sample mean, \bar{x} , and an unbiased estimate, s_X^2 , of the population variance, σ_X^2 , with the following results.

$$\bar{x} = 17.51 \quad s_X^2 = 1.273$$

- (a) Test, at the 10% significance level, the claim that the mean weight of adult male dormice in this population in June is 17 grams. *(7 marks)*
- (b) The conservationist takes another random sample of 9 adult male dormice in October, just before the hibernation period, and records the weight, y grams, of each dormouse. He calculates the sample mean, \bar{y} , and an unbiased estimate, s_Y^2 , of the population variance, σ_Y^2 , giving

$$\bar{y} = 31.81 \quad \text{and} \quad s_Y^2 = 1.719.$$

- (i) Assuming that $\sigma_X^2 = \sigma_Y^2$, calculate a 95% confidence interval for the mean increase in weight of adult male dormice between June and October. *(8 marks)*
- (ii) The conservationist believes that, to prepare for hibernation, adult male dormice increase their weight by at least 75%, on average, between June and October. Use your confidence interval to comment on his belief. *(2 marks)*

END OF QUESTIONS