General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP6**
Unit Pure 6

ASSESSMENT and QUALIFICATIONS ALLIANCE

Friday 16 January 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP6.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The position vectors of three points A, B and C relative to an origin O are given by

$$\begin{aligned} \mathbf{a} &= 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}, \\ \mathbf{b} &= 2\mathbf{i} + 3\mathbf{j}, \\ \text{and} \quad \mathbf{c} &= \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} \end{aligned}$$

respectively.

(a) Find:

(i) $\mathbf{a} \times \mathbf{b}$; *(2 marks)*

(ii) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$. *(2 marks)*

(b) State a geometrical relationship between the points O, A, B and C. *(1 mark)*

2 (a) Find the value of the determinant

$$\begin{vmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \\ 0 & -1 & 2 \end{vmatrix}.$$ *(2 marks)*

(b) Determine whether the vectors

$$\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}$$

are linearly dependent. *(1 mark)*

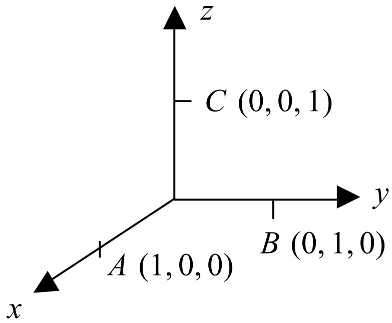
(c) Express the vector $\begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$ as a linear combination of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$. *(6 marks)*

3 A matrix $\mathbf{M}_1$ is given by

$$\mathbf{M}_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Give a geometrical description of the transformation T_1 represented by $\mathbf{M}_1$. *(2 marks)*

(b) The diagram below shows the points $A\ (1,0,0)$, $B\ (0,1,0)$ and $C\ (0,0,1)$.



A second transformation, T_2, is a rotation of π radians about the line $x = z$, $y = 0$.

(i) Find the images of the points A, B, and C under T_2. *(2 marks)*

(ii) Write down the matrix $\mathbf{M}_2$ which represents this transformation. *(2 marks)*

(c) (i) Show that the matrix $\mathbf{M}_3$, which represents the transformation T_2 followed by the transformation T_1, is given by

$$\mathbf{M}_3 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ *(2 marks)*

(ii) Give a geometrical description of the transformation represented by the matrix $\mathbf{M}_3$. *(2 marks)*

TURN OVER FOR THE NEXT QUESTION

4 The planes Π_1 and Π_2 have equations

$$x + 2y - z = 1$$

and

$$x + 3y + z = 6$$

respectively.

(a) Verify that the point P, with coordinates $(1, 1, 2)$, lies on both planes. *(1 mark)*

(b) Find the equation of l, the line of intersection of Π_1 and Π_2, giving your answer in the form $\dfrac{x-\alpha}{p} = \dfrac{y-\beta}{q} = \dfrac{z-\gamma}{r}$. *(5 marks)*

(c) Find the acute angle between the line l and the line through P which is parallel to the y-axis. Give your answer to the nearest 0.1 of a degree. *(3 marks)*

5 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & p \\ 0 & -5 & p \end{bmatrix}$$

and

$$\mathbf{B} = \begin{bmatrix} p & -1 \\ -2 & 0 \\ -3 & 3 \end{bmatrix}.$$

(a) Find **AB** in terms of p. *(3 marks)*

(b) Given that $\det \mathbf{AB} = 0$:

(i) find the possible values of p; *(4 marks)*

(ii) for each value of p, find the matrix $\mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}$; *(3 marks)*

(iii) explain why $(\mathbf{AB})^{-1}$ does not exist. *(1 mark)*

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}.$$

(a) Verify that $\mathbf{M}$ has an eigenvalue 1 and find a corresponding eigenvector. *(5 marks)*

(b) (i) Show that $\mathbf{M}$ has only one other distinct eigenvalue and find this eigenvalue. *(4 marks)*

(ii) Deduce that any non-zero vector of the form $\begin{bmatrix} p \\ q \\ q \end{bmatrix}$, where p and q are real, is an eigenvector corresponding to this eigenvalue. *(3 marks)*

(c) (i) Find a Cartesian equation of a line l_1 lying in the plane $x = 0$ such that each point of l_1 is invariant under the transformation T represented by $\mathbf{M}$. *(2 marks)*

(ii) Find a Cartesian equation of another line l_2 in the plane $x = 0$ which is invariant under T. *(2 marks)*

END OF QUESTIONS

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