General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) **MAP3**
Unit Pure 3

ASSESSMENT and QUALIFICATIONS ALLIANCE

Friday 23 January 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A curve is given by the parametric equations

$$x = 3t^2, \qquad y = 6t.$$

(a) (i) Find $\dfrac{dy}{dx}$ in terms of t. *(2 marks)*

(ii) Find the gradient of the curve at the point where $t = \frac{1}{2}$. *(1 mark)*

(b) (i) Find the equation of the curve in the form $x = f(y)$. *(2 marks)*

(ii) Find $\dfrac{dx}{dy}$ in terms of y and hence verify your answer to part (a)(ii). *(4 marks)*

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{1 + x^3}.$$

Starting at the point $(1, 0.5)$ on the curve, use a step-by-step method with a step length of 0.25 to estimate the value of y at $x = 1.5$, giving your answer to two decimal places. *(5 marks)*

3 A microbiologist is studying the growth of populations of simple organisms.

For one such organism, the model proposed is

$$P = 100 - 50e^{-\frac{1}{4}t},$$

where P is the population after t minutes.

(a) Write down:

(i) the initial value of the population; *(1 mark)*

(ii) the value which the population approaches as t becomes large. *(1 mark)*

(b) Find the time at which the population will have a value of 75, giving your answer to two significant figures. *(4 marks)*

4 (a) Express $\dfrac{8+3x}{(1+3x)(2-x)}$ in the form $\dfrac{A}{1+3x}+\dfrac{B}{2-x}$. *(3 marks)*

(b) Obtain the first three terms in the expansion of $\dfrac{1}{1+3x}$ in ascending powers of x. *(2 marks)*

(c) Show that the first three terms in the expansion of $\dfrac{1}{2-x}$ in ascending powers of x are $\dfrac{1}{2}+\dfrac{x}{4}+\dfrac{x^2}{8}$. *(3 marks)*

(d) Hence, or otherwise, obtain the first three terms in the expansion of $\dfrac{8+3x}{(1+3x)(2-x)}$ in ascending powers of x. *(3 marks)*

(e) State the range of values of x for which the expansion in part (d) is valid. *(2 marks)*

5 (a) (i) The function f is given by

$$f(x) = e^{-2x}.$$

By differentiation, find $f'(x)$ and $f''(x)$. *(2 marks)*

(ii) Hence show that the first three terms in the Maclaurin series of $f(x)$ are

$$1 - 2x + 2x^2.$$ *(2 marks)*

(b) (i) Using the approximation $\cos x \approx 1 - \dfrac{x^2}{2}$, write down a similar approximation for $\cos 3x$. *(1 mark)*

(ii) Use your results from parts (a)(ii) and (b)(i) to find an approximate solution of the equation

$$e^{-2x} = \cos 3x \quad \text{for} \quad 0 < x < 1.$$ *(3 marks)*

Turn over ▶

6 The speed v m s^{-1} of a pebble falling through still water after t seconds can be modelled by the differential equation

$$\frac{dv}{dt} = 10 - 5v.$$

A pebble is placed carefully on the surface of the water at time $t = 0$ and begins to sink.

(a) Show that $t = \frac{1}{5}\ln\left(\frac{2}{2 - v}\right)$. *(6 marks)*

(b) Use the model to find the speed of the pebble after 0.5 seconds, giving your answer to two significant figures. *(3 marks)*

7 The points A and B have coordinates $(3, -1, 2)$ and $(5, 3, -2)$ respectively.

(a) (i) Find the distance between A and B. *(2 marks)*

(ii) Find the coordinates of M, where M is the mid-point of AB. *(1 mark)*

(b) The point C has coordinates $(8, -2, -1)$.

Show that $\overrightarrow{CM}$ is perpendicular to $\overrightarrow{AB}$. *(2 marks)*

(c) Points A and B both lie in a plane Π.

Given that the line CM is perpendicular to the plane Π, show that the Cartesian equation of Π is

$$4x - 3y - z = 13.$$ *(2 marks)*

(d) The line CP has equation $\mathbf{r} = \begin{pmatrix} 8 \\ -2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \\ 3 \end{pmatrix}$ and intersects Π at the point P.

Find the coordinates of the point P. *(3 marks)*

END OF QUESTIONS