General Certificate of Education
January 2004
Advanced Level Examination

MATHEMATICS (SPECIFICATION A) MAP2
Unit Pure 2

Monday 19 January 2004 Morning Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a standard scientific calculator **only**.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) The quadratic equation $2x^2 - 6x + 1 = 0$ has roots α and β.

Write down the numerical values of:

(i) $\alpha\beta$; *(1 mark)*

(ii) $\alpha + \beta$. *(1 mark)*

(b) Another quadratic equation has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Find the numerical values of:

(i) $\frac{1}{\alpha} \times \frac{1}{\beta}$; *(1 mark)*

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$. *(2 marks)*

(c) Hence, or otherwise, find the quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, writing your answer in the form $x^2 + px + q = 0$. *(2 marks)*

2 A circle has equation

$$x^2 + y^2 - 4x + 4y - 12 = 0.$$

(a) Find:

(i) the coordinates of the centre of the circle;

(ii) the radius of the circle. *(5 marks)*

(b) Find the coordinates of the **two** points where the circle crosses the x-axis. *(3 marks)*

(c) Find the equation of the tangent to the circle at the point $(4, 2)$. *(4 marks)*

3 (a) Find the value of $\tan^{-1} 2.4$, giving your answer in radians to three decimal places. *(1 mark)*

(b) Express $10\sin\theta + 24\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. *(3 marks)*

(c) Hence:

(i) write down the maximum value of $10\sin\theta + 24\cos\theta$; *(1 mark)*

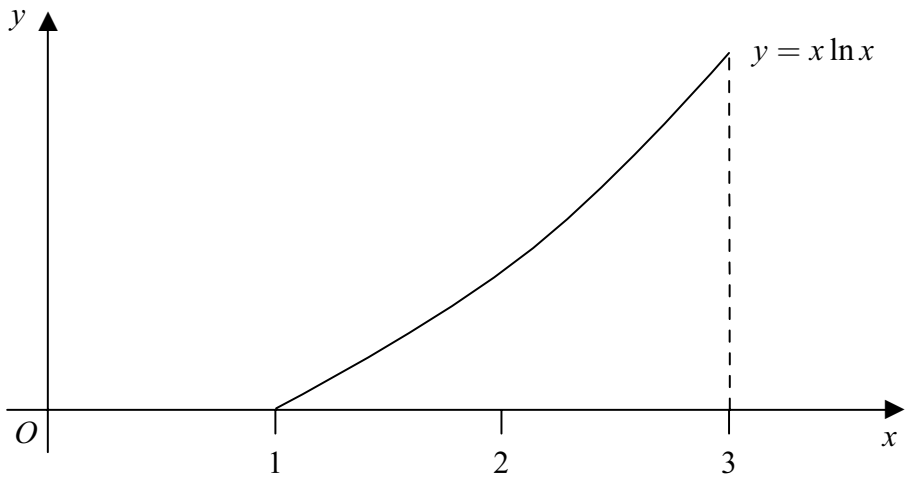
(ii) find a value of θ at which this maximum value occurs. *(2 marks)*

4 (a) By using the chain rule, or otherwise, find $\frac{dy}{dx}$ when $y = \ln(x^2 + 9)$. *(3 marks)*

(b) Hence show that $\int_0^3 \frac{x}{x^2 + 9}\,dx = \frac{1}{2}\ln 2$. *(3 marks)*

(c) Show that $\int_0^3 \frac{x + 1}{x^2 + 9}\,dx = \frac{1}{2}\ln 2 + \frac{\pi}{12}$. *(4 marks)*

5 The diagram below shows the graph of $y = x\ln x$ for $1 \leqslant x \leqslant 3$.

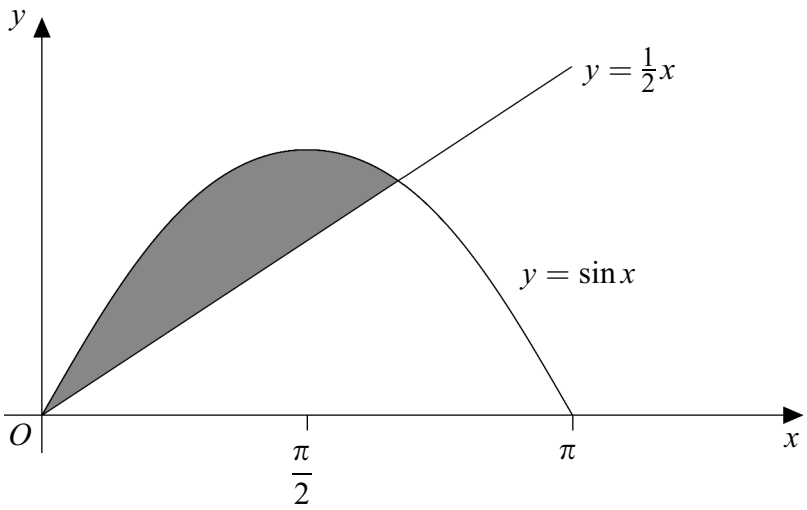


(a) Use the trapezium rule with four strips to find an approximation to the area enclosed by the graph of $y = x\ln x$ and the x-axis between $x = 1$ and $x = 3$. *(4 marks)*

(b) (i) Differentiate $2x^2\ln x - x^2$ with respect to x. *(3 marks)*

(ii) Hence, or otherwise, evaluate $\int_1^3 x\ln x\,dx$. *(3 marks)*

6 The diagram below shows the graphs of $y = \sin x$ and $y = \frac{1}{2}x$, for $0 \leqslant x \leqslant \pi$.



(a) Show that the equation $\sin x - \frac{1}{2}x = 0$ has a root in the interval $1 \leqslant x \leqslant 2$, where x is measured in radians. *(2 marks)*

(b) (i) Given that $f(x) = \sin x - \frac{1}{2}x$, find $f'(x)$. *(1 mark)*

(ii) Use a single application of the Newton–Raphson method, with an initial value $x = 2$, to show that the root of the equation $\sin x - \frac{1}{2}x = 0$ in the interval $1 \leqslant x \leqslant 2$ is approximately 1.9. *(3 marks)*

(c) (i) Show that $\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$. *(2 marks)*

(ii) Hence find $\int_0^{1.9} \sin^2 x \, dx$. *(1 mark)*

(d) The shaded region enclosed by the graph of $y = \sin x$ and the line $y = \frac{1}{2}x$ is rotated through one revolution about the x-axis to form a solid.

Calculate an approximation for the volume of this solid, giving your answer to two significant figures. *(5 marks)*

END OF QUESTIONS