ASSESSMENT and QUALIFICATIONS ALLIANCE

Mark scheme January 2004

GCE

Mathematics A

Unit MAP2

Key to mark scheme

M	mark is for	method
m	mark is dependent on one or more M marks and is for	method
A	mark is dependent on M or m mark and is for	accuracy
B	mark is independent of M or m marks and is for	method and accuracy
E	mark is for	explanation
√ or ft or F		follow through from previous incorrect result
CAO		correct answer only
AWFW		anything which falls within
AWRT		anything which rounds to
AG		answer given
SC		special case
OE		or equivalent
A2,1		2 or 1 (or 0) accuracy marks
– x EE		Deduct x marks for each error
NMS		No method shown
PI		Perhaps implied
c		Candidate

Abbreviations used in marking

MC – x	deducted x marks for miscopy
MR – x	deducted x marks for misread
ISW	ignored subsequent working
BOD	gave benefit of doubt
WR	work replaced by candidate

Application of mark scheme

Correct answer without working	mark as in scheme
Incorrect answer without working	zero marks unless specified otherwise

Award method and accuracy marks as appropriate to an alternative solution using a correct method or partially correct method.

Q	Solution	Marks	Total	Comments
1 (a)(i)	$\alpha\beta = \frac{1}{2}$	B1		
(ii)	$\alpha + \beta = 3$	B1	2	
(b)(i)	$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = 2$	B1$\surd$	1	
(ii)	$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = 6$	M1A1$\surd$	2	
(c)	$x^2 - (sum)x + (product) = 0$ $x^2 - 6x + 2 = 0$	M1 A1$\surd$	2	Replace x by $\frac{1}{x}$ $2\left(\frac{1}{x}\right)^2 - 6\left(\frac{1}{x}\right) + 1 = 0$ $\frac{2}{x^2} - \frac{6}{x} + 1 = 0$ × by x^2 to give $x^2 - 6x + 2 = 0$
	Total		**7**	

Q	Solution	Marks	Total	Comments
2 (a)(i)	Centre $(2\,,\,-2)$	B1		
(ii)	Complete the square $(x-2)^2+(y+2)^2=20$ $\therefore r^2=20$ $r=\sqrt{20}$ or (AWRT 4.47)	M1 A1 A1 A1$\surd$	5	Attempted LHS correct RHS correct (on their RHS > 0)
(b)	Crosses x-axis when $y=0$ $\therefore x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ $\therefore$ crosses x-axis at the points $(6\,,\,0)$ & $(-2\,,\,0)$	M1 m1 A1	3	For use of $y=0$ For solving quadratic by any correct method attempted Accept $x=6$ *and* $x=-2$ if $y=0$ used
(c)	Slope of radius = $\dfrac{2--2}{4-2}=\dfrac{4}{2}=2$ Use $m_1m_2=-1$ for perpendicular lines $\therefore$ slope of tangent = $-\dfrac{1}{2}$ Equation of tangent is $y-2=-\dfrac{1}{2}(x-4)$ $2y-4=-x+4$ $x+2y-8=0$	B1$\surd$ B1$\surd$ M1 A1$\surd$	4	On their centre On their slope of radius If $m_1m_2=-1$ used then: use of $y-y_1=m(x-x_1)$ or any other correct method Accept any simplified form (on their value of m)
	Total		**12**	

Q	Solution	Marks	Total	Comments
3 (a)	$\beta = \tan^{-1}(2.4) = 1.176°$	B1	1	
(b)	$10\sin\theta + 24\cos\theta \equiv R\sin(\theta+\alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$			
	$R\sin\alpha = 24$ $R\cos\alpha = 10$			
		M1		Any correct attempt at finding R or α
	$\tan\alpha = 2.4$ $\quad \therefore \alpha = 1.176°$	A1		Correct α (AWRT 1.18)
	$R^2 = 24^2 + 10^2 = 676$ $\quad R = 26$	A1		Correct R
	$\Rightarrow 26\sin(\theta + 1.176)$		3	
(c)(i)	Maximum value = 26	B1√	1	On their answer to part (b) $(\pm 26 \text{ gets B0})$ (based on a valid method used in (b))
(ii)	$\sin(\theta + 1.176) = 1$	M1		
	$\therefore \theta + 1.176 = \frac{\pi}{2}$ $\theta = 0.395°$	A1√	2	On their value of α (6.68, 13.0, ……….)
	Total		**7**	

Q		Solution	Marks	Total	Comments
4	(a)	$y=\ln\left(x^2+9\right)$			
		let $u=x^2+9$ then $\dfrac{\mathrm{d}u}{\mathrm{d}x}=2x$ and $y=\ln u \quad \therefore \dfrac{\mathrm{d}y}{\mathrm{d}u}=\dfrac{1}{u}=\dfrac{1}{x^2+9}$	M1		
		$\therefore \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}u}\times\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{x^2+9}\times 2x$	M1		Use of chain rule
		$=\dfrac{2x}{x^2+9}$	A1	3	CAO
	(b)	$\displaystyle\int_0^3 \frac{x}{x^2+9}\,\mathrm{d}x=\left[\frac{1}{2}\ln\left(x^2+9\right)\right]_0^3$	M1		
		$=\dfrac{1}{2}\ln 18-\dfrac{1}{2}\ln 9$	A1		
		$=\dfrac{1}{2}\ln 2$	A1	3	AG
	(c)	$\displaystyle\int_0^3 \frac{x+1}{x^2+9}\,\mathrm{d}x=\int_0^3 \frac{x}{x^2+9}\,\mathrm{d}x+\int_0^3 \frac{1}{x^2+9}\,\mathrm{d}x$	M1		Attempted
		$=\dfrac{1}{2}\ln 2+\dfrac{1}{3}\left[\tan^{-1}\left(\dfrac{x}{3}\right)\right]_0^3$	A1		
		$==\dfrac{1}{2}\ln 2+\dfrac{1}{3}\left[\tan^{-1}(-1)-\tan^{-1}(0)\right]$	M1		Limits used in correct expression
		$=\dfrac{1}{2}\ln 2+\dfrac{\pi}{12}$	A1	4	AG
		Total		**10**	

Q	Solution	Marks	Total	Comments
5 (a)	$y = x\ln x$ $y(1) = 0$ $y(1.5) = 0.60820$ $y(2) = 1.38629$ $y(2.5) = 2.29073$ $y(3) = 3.29584$	B2		B1 for any two correct B2 for all correct
	$Area = \frac{1}{2}\times\frac{1}{2}\times(0+3.2958+2[4.2852])$	M1		
	$= 2.97$	A1	4	2.96657
(b)(i)	$2x^2\times\frac{1}{x}+(\ln x)\times 4x-2x$ $= 4x\ln x$	M1A1 A1	3	Product rule attempted
(ii)	$\int_1^3 x\ln x\,dx = \frac{1}{4}\left[2x^2\ln x - x^2\right]_1^3$	M1		
	$= \frac{1}{4}(\{18\ \ln 3-9\}-\{-1\})$	M1		
	$= \frac{1}{4}(18\ \ln 3-8)$ $(= 2.94)$	A1	3	(2.943755)
	Total		**10**	

Q	Solution	Marks	Total	Comments
6 (a)	$\mathrm{f}(1)=0.341$ $\mathrm{f}(2)=-0.091$	M1		
	Change of sign $\Rightarrow$ $\therefore$ root in the interval $1 \le x \le 2$	A1	2	
(b)(i)	$\mathrm{f}'(x)=\cos x-\frac{1}{2}$	B1	1	
(ii)	$x_{n+1}=x_n-\frac{\mathrm{f}(x)}{\mathrm{f}'(x_n)}=x_n-\frac{\sin x_n-\frac{1}{2}x_n}{\cos x_n-\frac{1}{2}}$	M1		N-R formula used
	$x_0=2 \quad \therefore \quad x_1=2-\frac{\sin 2-1}{\cos 2-\frac{1}{2}}$	m1		Radians used in correct formula
	$x_1=1.901\approx 1.9$	A1	3	AG
(c)(i)	$\sin^2 x=\frac{1}{2}(1-\cos 2x)$ $\therefore \quad \int \sin^2 x\,\mathrm{d}x=\frac{1}{2}\int(1-\cos 2x)\,\mathrm{d}x$	M1		
	$=\frac{1}{2}x-\frac{1}{4}\sin 2x+c$	A1	2	AG
(ii)	$\int_0^{1.9}\sin^2 x=\left[\frac{1}{2}x-\frac{1}{4}\sin 2x\right]_0^{1.9}=1.10$	B1	1	
(d)	Volume of solid formed $=V_1-V_2$	M1		
	$V_1=\pi\int_0^{1.90}\sin^2 x\,\mathrm{d}x$ $=\pi\times 1.10$ $(=3.47)$	M1		for V_1 (3.46507) allow 3.46 ($1.10\times\pi$)
	$V_2=\frac{1}{3}\times\pi\times(0.95)^2\times 1.90$ **or** $\pi\int_0^{1.9}\left(\frac{1}{2}x\right)^2\mathrm{d}x$ $(=1.796)$	M1		for V_2
	$\therefore$ Volume of solid formed = 1.67	A1		(1.66938) allow 1.66
	Volume = 1.7 (2sf)	A1	5	
	Total		**14**	
	Total		**60**	