AQA

ASSESSMENT and
QUALIFICATIONS
ALLIANCE

MATHEMATICS (SPECIFICATION A)   **MAP1**
Unit Pure 1

Wednesday 14 January 2004   Morning Session

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MAP1

Answer **all** questions.

1 (a) Find $\int x^{\frac{1}{2}}\,\mathrm{d}x$. *(2 marks)*

(b) Hence find the value of $\int_{0}^{2} x^{\frac{1}{2}}\,\mathrm{d}x$,

giving your answer in the form $p\sqrt{2}$, where p is a rational number. *(3 marks)*

2 The nth term of a geometric sequence is u_n, where
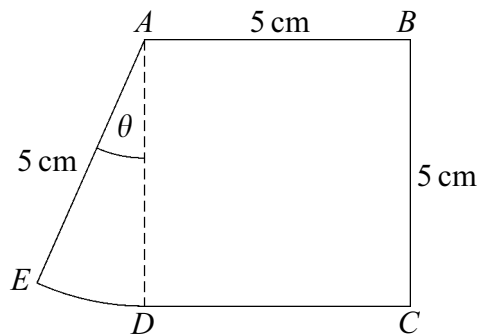
$$u_n = 2 \times 3^n.$$

(a) Find the values of u_1 and u_2. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) Show that the sum of the first 10 terms is $3(3^{10} - 1)$. *(3 marks)*

3 The diagram shows a shape $ABCDE$. The shape consists of a square $ABCD$, with sides of length 5 cm, and a sector ADE of a circle with centre A and radius 5 cm. The angle of the sector is θ radians.

(a) Find the area of the sector ADE in terms of θ. *(2 marks)*

(b) The area of the sector ADE is a quarter of the area of the square $ABCD$.

(i) Find the value of θ. *(2 marks)*

(ii) Find the perimeter of the shape $ABCDE$. *(2 marks)*

4 (a) An arithmetic sequence has first term 100 and common difference 2.

 (i) Write down the second and third terms. *(2 marks)*

  (ii) Given that the last term is 200, find the number of terms. *(3 marks)*

 (b) A tape dispenser has a length of tape wrapped round a circular cylinder. The length of tape in the first layer (nearest to the cylinder) is 100 mm. Each further layer is 2 mm longer than the one before. The outer layer has 200 mm of tape.

  Calculate the total length of tape. *(3 marks)*

5 The diagram shows a sketch of the graph of

$$y = e^{2x} + 2x^{-1} \qquad \text{for } x > 0.$$
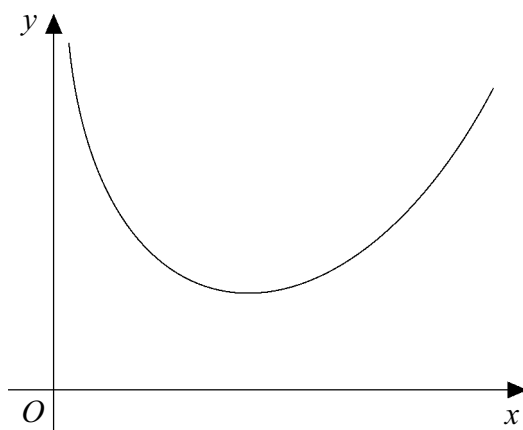
 (a) Find $\dfrac{dy}{dx}$. *(3 marks)*

 (b) Show that, at the stationary point on the graph, $x^2 e^{2x} = 1$. *(3 marks)*

 (c) Deduce that, at the stationary point,

$$x e^x = 1$$

  and hence

$$\ln x + x = 0. \qquad\qquad \textit{(3 marks)}$$

 (d) Show that the equation

$$\ln x + x = 0$$

  has a root between 0.5 and 0.6. *(3 marks)*

 (e) Find $\displaystyle\int (e^{2x} + 2x^{-1})dx$. *(3 marks)*

Turn over ▶

6 (a) The functions f and g are defined by:
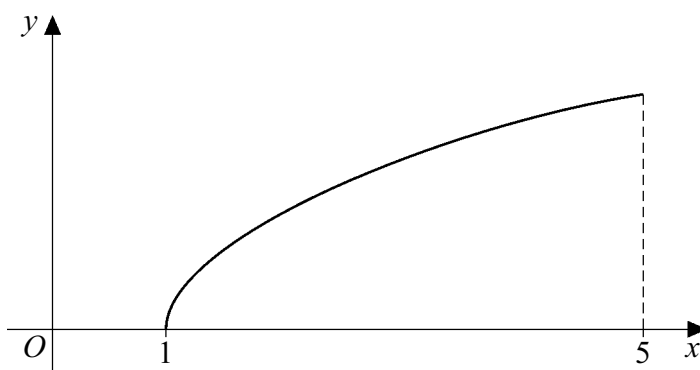
$$f(x) = \sqrt{x} \qquad \text{for } x \geqslant 0;$$

$$g(x) = x - 1 \qquad \text{for all values of } x.$$

(i) Write down expressions for $fg(x)$ and $gf(x)$. *(2 marks)*

(ii) Verify that

$$x = 1 \Rightarrow fg(x) = gf(x).$$ *(1 mark)*

(b) The diagram shows the graph of $y = h(x)$, where the function h is defined **for the domain $1 \leqslant x \leqslant 5$** by

$$h(x) = \sqrt{x - 1}.$$

(i) Describe the transformation by which the graph of $y = \sqrt{x - 1}$ can be obtained from the graph of $y = \sqrt{x}$. *(2 marks)*

(ii) Write down the range of the function h. *(1 mark)*

(iii) Write down the domain and range of the inverse function h^{-1}. *(2 marks)*

(iv) Find an expression for $h^{-1}(x)$. *(3 marks)*

7 (a) Write down the exact values of $\sin \dfrac{\pi}{6}$, $\cos \dfrac{\pi}{6}$ and $\tan \dfrac{\pi}{6}$. *(3 marks)*

(b) It is given that x satisfies the equation

$$3 \sin^2 x = \cos^2 x.$$

By first using an appropriate trigonometrical identity to simplify this equation, find all the solutions of the equation in the interval $0 \leqslant x \leqslant 2\pi$. *(6 marks)*

END OF QUESTIONS