General Certificate of Education
January 2004
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS (SPECIFICATION A) **MAM3**
Unit Mechanics 3

Tuesday 27 January 2004 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAM3.
- Answer **all** questions.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$ unless otherwise stated.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.

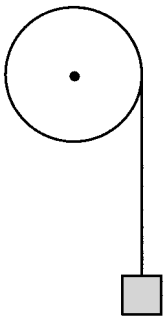
Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 A heavy uniform flywheel is free to rotate about a smooth fixed horizontal axis through its centre. The radius of the flywheel is 0.2 m and its moment of inertia about the axis is 10 kg m^2.

A light cord is wrapped tightly round the circumference of the flywheel. A mass of 5 kg is suspended from the free end of the cord.

The system is initially at rest and then released. The cord does not slip during the subsequent motion.



When the flywheel has turned through one complete revolution:

(a) calculate the distance that the 5 kg mass has dropped; *(2 marks)*

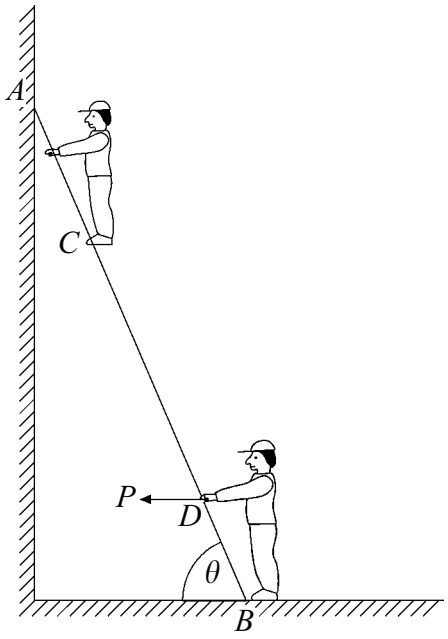
(b) using the principle of conservation of energy, determine the angular speed of the flywheel. *(6 marks)*

2 A uniform ladder AB, of weight W and length $4a$, is resting with A against a smooth vertical wall and B on rough horizontal ground.

A man of weight $4W$ is standing at a point C on the ladder, where $BC = 3a$. The force exerted by the man's hands on the ladder is negligible and the centre of mass of the man is vertically above C.

Another man stands on the ground near the bottom of the ladder and pushes on the ladder at the point D, where $BD = a$. The force applied at D is horizontal, of magnitude P and is just sufficient to prevent the ladder sliding away from the wall.

The ladder is inclined at an angle $\theta = \tan^{-1}\left(\frac{12}{5}\right)$ to the horizontal.



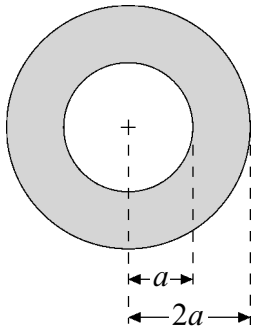
The coefficient of friction between the base of the ladder and the ground is $\frac{1}{4}$.

(a) Show that the horizontal frictional force at B is of magnitude $\frac{5W}{4}$. *(3 marks)*

(b) Find the value of P in terms of W. *(6 marks)*

TURN OVER FOR THE NEXT QUESTION

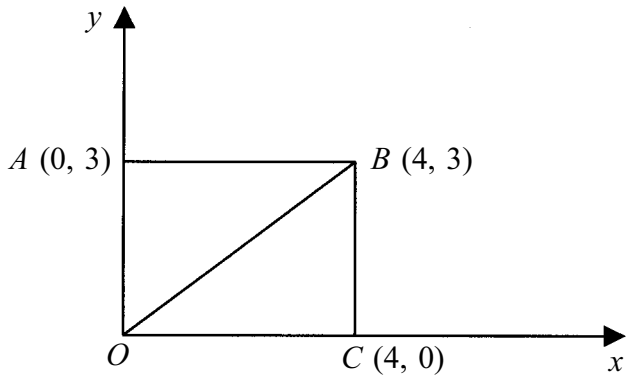
3 A thin, uniform, circular disc of radius $2a$ has a second disc of radius a cut from its centre. The resulting ring has mass M and is shown shaded on the diagram.



(a) Show, by integration, that the moment of inertia of the ring about an axis through its centre and perpendicular to the plane of the ring is $\frac{5Ma^2}{2}$. *(7 marks)*

(b) Hence determine the moment of inertia of the ring about an axis along a diameter. *(3 marks)*

4 The points O, A, B and C have coordinates (0, 0), (0, 3), (4, 3) and (4, 0) respectively.



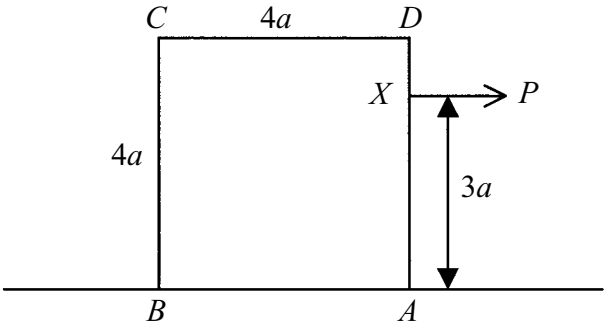
A clockwise couple of magnitude 19 N m acts in the plane together with forces of magnitudes 5 N, 6 N, 4 N, 7 N and 5 N acting along OA, BA, CB, OC, and OB respectively.

(a) Show that the resultant of this system of forces and the couple is of magnitude 13 N. *(5 marks)*

(b) (i) Show that the line of action of the resultant cuts the y-axis at $(0, -3)$. *(5 marks)*

(ii) Find the equation of the line of action of the resultant. *(3 marks)*

5 A uniform cube, of side $4a$ and weight W, rests on a horizontal rough plane surface. The diagram shows a vertical cross-section $ABCD$ parallel to opposite faces of the cube and through its centre of mass.



A horizontal force P is applied to AD at a point X where $AX = 3a$. The coefficient of friction between the cube and the plane is μ.

(a) In the case where the cube remains upright but slides along the plane under the action of P, determine an inequality for P in terms of W and μ. *(3 marks)*

(b) In the case where the cube does not slide under the action of P but is on the point of toppling about the edge through A:

(i) draw a diagram showing the lines of action of the forces on the cube; *(1 mark)*

(ii) determine the value of P in terms of W. *(2 marks)*

(c) The value of P is gradually increased from zero. If $\mu = 0.6$, determine whether the cube will slide before it topples. *(3 marks)*

TURN OVER FOR THE NEXT QUESTION

6 A uniform rod AB, of mass $3m$ and length $2l$, is lying at rest on a smooth horizontal table. The rod is free to rotate about a fixed vertical axis through A.

(a) Show that the moment of inertia of the rod about the axis through A is $4ml^2$. *(1 mark)*

(b) A particle of mass $5m$ is travelling with speed u across the table in a direction perpendicular to the rod. The particle strikes the rod at its midpoint.

(i) The collision between the particle and the rod is perfectly elastic. Immediately after the collision the angular speed of the rod is ω and the speed of the particle is v. Show that

$$l\omega = u + v. \qquad \textit{(2 marks)}$$

(ii) By considering the angular momentum of the system, show that

$$v = \frac{u}{9}. \qquad \textit{(6 marks)}$$

(iii) State the direction of motion of the particle immediately after the collision. *(1 mark)*

(c) Express ω in terms of u and l. *(1 mark)*

END OF QUESTIONS