General Certificate of Education
January 2004
Advanced Subsidiary Examination

MATHEMATICS (SPECIFICATION A)   **MAD1**
Unit Discrete 1

Monday 12 January 2004   Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Questions 6 and 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 20 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MAD1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- Tie loosely any additional sheets you have used, including the insert for use in Questions 6 and 7, to the back of your answer book before handing it to the invigilator.

Information
- The maximum mark for this paper is 60.
- Mark allocations are shown in brackets.
- Further copies of the insert for use in Questions 6 and 7 are available on request.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

MAD1

Answer **all** questions.

1 Six people A, B, C, D, E and F are to be matched to six tasks 1, 2, 3, 4, 5 and 6. The following table shows the tasks that each of the people is able to undertake.

Person	Task(s)
A	1, 2, 3
B	1, 2, 5
C	2, 3, 4
D	3, 4, 6
E	3
F	2

(a) Show this information on a bipartite graph.   *(2 marks)*

(b) The following is the initial matching:

$$A - 2, \quad B - 1, \quad C - 3, \quad D - 4.$$

Demonstrate, by using an algorithm from this initial matching, how each person can be allocated a task that they can undertake.   *(5 marks)*

2 The following diagram shows a network of roads connecting six towns. The number on each arc represents the distance, in miles, between towns. The road connecting towns D and F has length x miles, where $x < 13$.

B

13   14

A   12   C

11

10   9

F   x   D

6   7

E
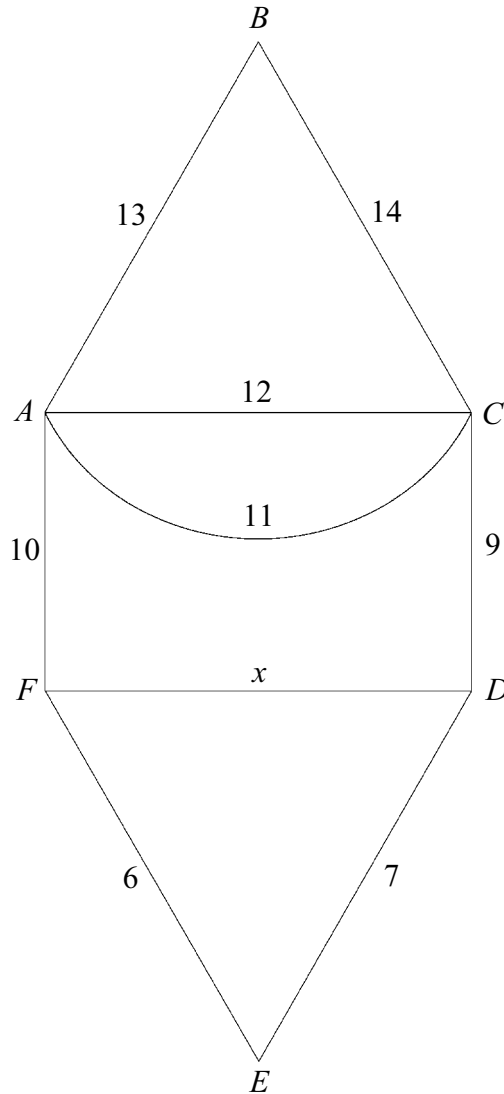
An optimal Chinese Postman route, starting and finishing at A, has length 100 miles. Find the value of x. *(4 marks)*

3 The following algorithm is to be used on different sets of numbers.

> Line 10   INPUT X, Y
>
> Line 20   LET $A = Y$
>
> Line 30   LET $B = 0$
>
> Line 40   LET $A = A - X$
>
> Line 50   LET $B = B + 1$
>
> Line 60   IF $A \geqslant X$ THEN GOTO Line 40
>
> Line 70   PRINT A, B
>
> Line 80   STOP

(a) Trace the algorithm:

   (i)   in the case when $X = 5$ and $Y = 20$;   *(2 marks)*

   (ii) in the case when $X = 7$ and $Y = 29$.   *(3 marks)*

(b) State the purpose of the algorithm.   *(2 marks)*

4 Tony is going on a touring holiday in America. The following table shows the five cities that he is to visit. The figures represent the distances, in miles, between the cities.

	Los Angeles (LA)	Las Vegas (LV)	Palm Springs (PS)	Santa Barbara (SB)	San Diego (SD)
Los Angeles (LA)	—	190	210	90	185
Las Vegas (LV)	190	—	180	140	300
Palm Springs (PS)	210	180	—	230	150
Santa Barbara (SB)	90	140	230	—	250
San Diego (SD)	185	300	150	250	—

Tony is to start his tour at Los Angeles, visiting each city once before returning to Los Angeles.

(a) Use the nearest neighbour algorithm, starting from Los Angeles, to find an upper bound for a tour of the five cities. *(4 marks)*

(b) By deleting Los Angeles, obtain a lower bound for a tour of the five cities. *(5 marks)*

(c) An optimal tour has length *T* miles. Using your answers to parts (a) and (b), write down a statement about *T*. *(1 mark)*

**5** A connected graph has five vertices and has arc lengths of

4, 7, 7, 7, 8, 8, 9 and 12 units.

(a) State the minimum length of a minimum spanning tree for any such graph. *(1 mark)*

(b) State the minimum length of a Hamiltonian cycle for any such graph. *(1 mark)*

(c) State the minimum length of an Eulerian cycle for any such graph. *(1 mark)*

(d) In the case when the length of its minimum spanning tree is 26 units, draw a sketch to show a possible graph. *(3 marks)*

Turn over ▶

6 [**Figure 1**, printed on the insert, is provided for use in answering this question.]

The following diagram shows motorways and main roads connecting towns in Sicily. The numbers represent the times taken, in minutes, to drive along each road. There are two airports on the island, one at Catania (C) and one at Palermo (P).
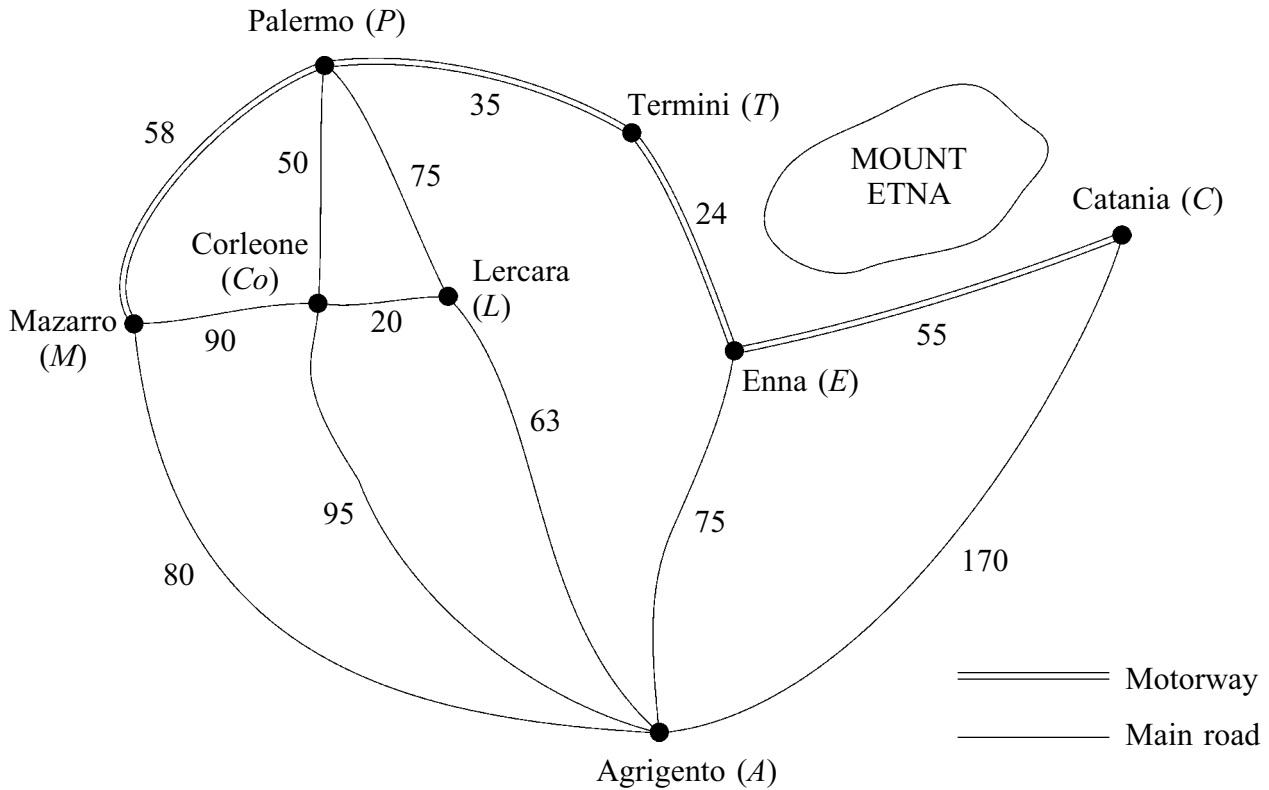
Stella plans to fly to Catania and then drive to Agrigento.

(a) Find, by inspection, the minimum time for Stella to drive from Catania to Agrigento.

(1 mark)

(b) Due to the volcano at Mount Etna erupting, Stella's flight is diverted to Palermo.

   (i) Use Dijkstra's algorithm on **Figure 1** to find the minimum time to drive from Palermo to Agrigento. *(5 marks)*

   (ii) State the route that she should take. *(1 mark)*

(c) Stella drives at 50 km/h on main roads and 100 km/h on motorways. Given that she keeps her driving time to a minimum, find the extra distance that she would have had to drive if she had landed at Catania airport rather than at Palermo airport. *(4 marks)*

7 [**Figure 2**, printed on the insert, is provided for use in answering this question.]

Every Saturday, David irons for the family. He irons three different types of clothing: dresses, shirts and jackets.

David knows that each Saturday:

he must iron at least 2 dresses, at least 5 shirts and at least 2 jackets;

he can only iron at most 28 items of clothing;

he takes 3 minutes to iron a dress, 2 minutes to iron a shirt and 2 minutes to iron a jacket. The iron must be used for at least 40 minutes but not more than 60 minutes.

Each Saturday, David irons x dresses, y shirts and z jackets.

(a) Find six inequalities that model David's situation. *(4 marks)*

(b) On a particular Saturday, David irons the same number of jackets as dresses.

David's aim is to iron the maximum number of items of clothing.

   (i) Show that the inequalities found in part (a) simplify to

   $$x \geqslant 2, \qquad y \geqslant 5, \qquad 2x + y \leqslant 28, \qquad 40 \leqslant 5x + 2y \leqslant 60.$$ *(2 marks)*

   (ii) On **Figure 2**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(7 marks)*

   (iii) Use your diagram to find how many of each type of clothing David might iron to achieve his aim. *(2 marks)*

END OF QUESTIONS