Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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11	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Use of Mathematics (Pilot) USE3

Mathematical Comprehension

Thursday 21 May 2015 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 45.

Advice
- You are advised to spend 1 hour on Section A and 30 minutes on Section B.
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Bacterial growth: friend or foe?** *on the Data Sheet.*

1 A colony of bacteria has a generation time of 15 minutes and, at a certain time, the number of bacteria cells is 4. How long will it take for the number of bacteria cells to reach 64?

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

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2 A colony of bacteria has a population, $N = 4500$ at a certain time.

(a) After 30 minutes the population has increased to $12\,000$. Calculate the generation time for this colony.

[3 marks]

(b) For this colony, use your answer to part **(a)** to state an exponential function of the form $N = N_0 e^{kt}$ that may be used to model its growth.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 The article shows that for exponential growth where the number of bacteria, N, is related to time, t, by a function of the form $N = N_0 e^{kt}$, $k = \frac{\ln 2}{G}$, where G is the generation time.

On the axes below:

(a) sketch a graph of k plotted against G;

[2 marks]

(b) interpret this graph, explaining how k varies with G.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

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4 The article implies that for a population that can be modelled as having exponential growth, the differential equation $\frac{\mathrm{d}N}{\mathrm{d}t} = kN$ applies. This can be solved to give $N = N_0 e^{kt}$, where $N = N_0$ when $t = 0$.

Show clearly all steps to arrive at this result.

[5 marks]

QUESTION PART REFERENCE

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5 For the colony of bacteria in the article, the growth data giving population density, P, at time, t, given in **Table 1**, is repeated below.

Table 1

Time, t minutes	**Population density, P**
0	2.2
16	3.6
32	6.0
48	10.1
64	16.9
80	22.7
96	36.0
112	51.0
128	70.4
144	82.7
160	92.8

(a) For this colony, the population density, P, can be approximated by $P = 2.2e^{0.029t}$.
Find the **instantaneous** growth rate at:

(i) $t = 16$;

(ii) $t = 96$.

[4 marks]

(b) Using the values from the table, calculate the average growth rate of the population density:

(i) at $t = 16$, using values for P between $t = 0$ and $t = 32$;

(ii) at $t = 96$, using values for P between $t = 80$ and $t = 112$.

[3 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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6 A scientist collects data in the laboratory for the growth of a colony of bacteria. She records how the population density, P, increases with time, t minutes, and attempts to fit a function $P = P_0 e^{kt}$ where P_0 is the population density at time $t = 0$.

The scientist plots a graph of $\ln P$ against t and finds it crosses the vertical axis at 3.045. She also finds that when $t = 0$, $\frac{dP}{dt} = 0.214$.

From this information, find P_0 and k.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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7 Data for another colony of bacteria is shown in the table below. The relative growth rate is also given. Use this to identify the range of times between which the conditions were favourable for an exponential model to be appropriate.
Explain your reasoning.

Time, t minutes	Population density, P	Growth rate	Relative growth rate
0	21	1	0.048
30	22	3	0.136
60	25	9	0.360
90	34	17	0.500
120	51	27	0.529
150	78	40	0.513
180	118	61	0.517
210	179	94	0.525
240	273	147	0.538
270	420	178	0.424
300	598		

[2 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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Section B

Answer **all** questions.

Answer each question in the space provided for that question.

Give me sunshine

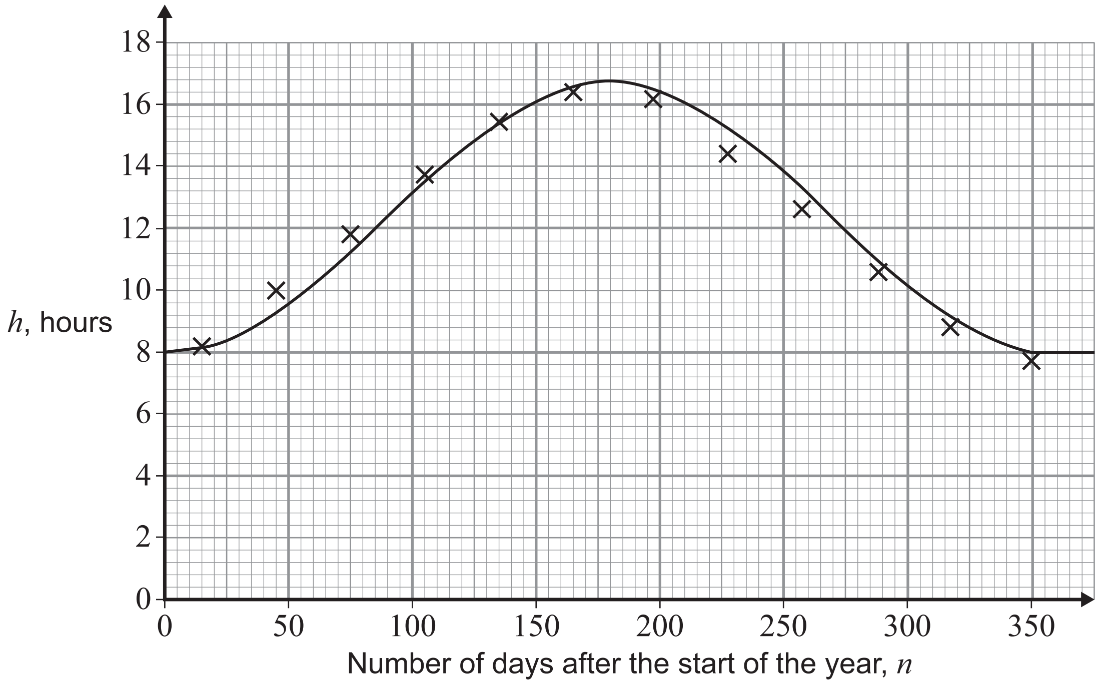
You will have noticed that the number of daylight hours per day varies considerably during the year in the UK. In summer months we have over 16 hours of daylight per day, whereas in winter months this reduces to fewer than 8 hours. This is because the Earth is tilted on its axis as it travels on its orbit around the sun.

A graph of data for the number of daylight hours per day, h, plotted against the number of days after the start of the year, n, for each month during a year in London, is shown in **Figure 7**.

The function $h = 12.4 + 4.4\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$ is also plotted in **Figure 7**.

This gives a relatively good approximation to the data.

Figure 7 Graph showing the number of daylight hours, h, plotted against the number of days after the start of the year, n, in London

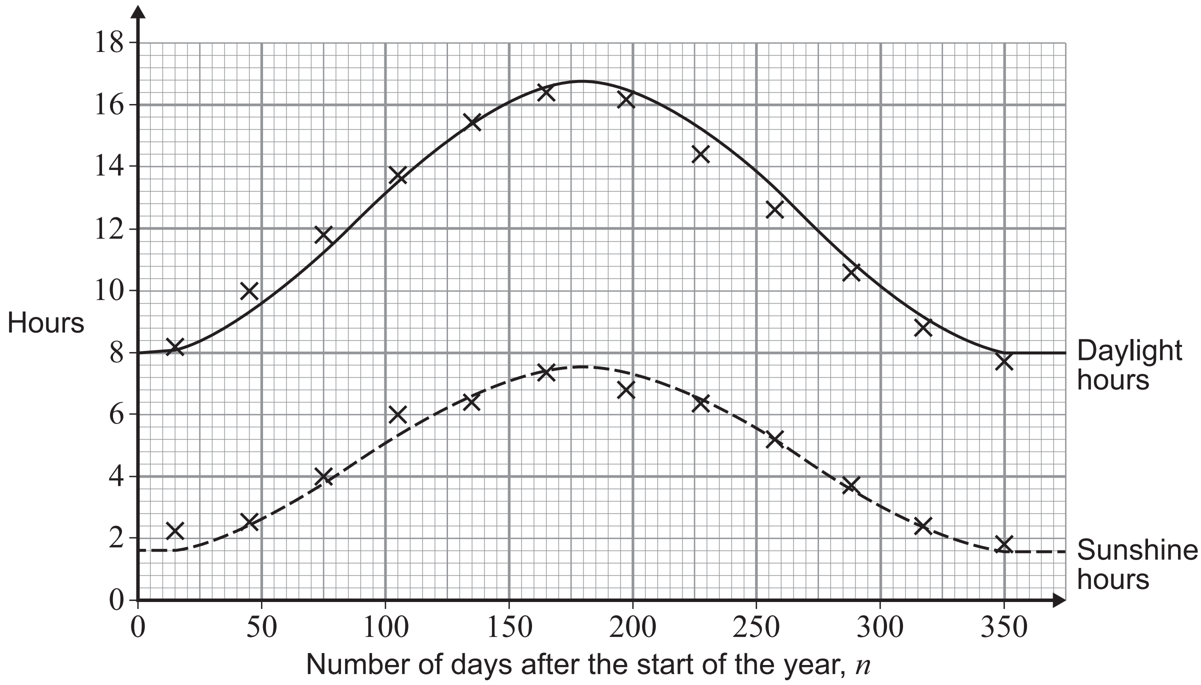


You might expect features of our climate, such as rainfall and sunshine hours, to also have similar underlying patterns which can be approximated by trigonometric functions.

Figure 8 shows further data for London. It shows data for the average number of sunshine hours per day, s, plotted against n. As suspected, a trigonometric function may be appropriate for modelling these data, and **Figure 8** confirms this by also plotting the function

$$s = 4.5 + 3\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$$

Figure 8 Graph showing data of how daylight hours, h, and sunshine hours, s, vary with number of days after the start of the year, n, in London



Functions such as that used to model the number of daylight hours per day can be useful, as they provide a quick and easy way to calculate values such as the number of daylight hours for any given day. For example, in the middle of January, $n = 15$, so

$$h = 12.4 + 4.4\sin\left(\frac{15\pi}{180} - \frac{\pi}{2}\right)$$
$$= 8.15$$

Such functions can also be used to provide other useful measures: for example, the average number of sunshine hours per day over a certain period, such as a month. To find such a measure, integration can be used. An approximation to the average number of sunshine hours per day in February ($31 \leqslant n \leqslant 59$) in London can be found using:

$$\bar{s}_{\text{Feb}} = \frac{1}{28}\int_{31}^{59} 4.5 + 3\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)\,\mathrm{d}n$$
$$= \frac{1}{28}\left[4.5n - \frac{3 \times 180}{\pi}\cos\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)\right]_{31}^{59}$$
$$= 2.40$$

As you can see from the graph in **Figure 8**, this seems a realistic answer, and would, for example, provide data that could be used in a travel guidebook.
Another use of functions is to find rates of change using differentiation. For example, differentiating, with respect to n, the function which models the number of daylight hours per day gives

$$\frac{\mathrm{d}h}{\mathrm{d}n} = \frac{4.4\pi}{180}\cos\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$$

$\frac{\mathrm{d}h}{\mathrm{d}n}$, that is the rate of change in the number of daylight hours per day, will have a maximum when $\cos\left(\frac{n\pi}{180} - \frac{\pi}{2}\right) = 1$. This first occurs when $\frac{n\pi}{180} - \frac{\pi}{2} = 0$, that is when $n = 90$. This can be confirmed by inspection of **Figure 8**.

Turn over ▶

8 The function, $h = 12.4 + 4.4\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$, can be used to model how the number of daylight hours per day, h, varies during a year in London. The function, $s = 4.5 + 3\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$, can be used to model how the number of sunshine hours per day, s, varies during a year in London.

(a) State the maximum number of daylight hours **and** the maximum number of sunshine hours that these functions predict will occur during a year in London.

[2 marks]

(b) Find n when these maximum values occur.
What date will this be?

[2 marks]

(c) Find the percentage of daylight hours that are sunshine hours for the value of n found in part **(b)**.

[2 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 8

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9 Using the method shown in the article for February, find the average number of sunshine hours per day in London for the 31 days in the month of March predicted by the function, $s = 4.5 + 3\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$.

[3 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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10 The average number of sunshine hours per day in London for the period $0 \leqslant n \leqslant 360$ predicted by the function $s = 4.5 + 3\sin\left(\frac{n\pi}{180} - \frac{\pi}{2}\right)$ is 4.5. Explain how you can deduce this result without using calculus.

[2 marks]

QUESTION PART REFERENCE

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11 Use calculus to find the value of n when the number of sunshine hours per day in London will be decreasing most rapidly.

[4 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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END OF QUESTIONS

Acknowledgement of copyright-holders and publishers

Question 7: David B. Fankhauser, Professor of Biology and Chemistry, http://biology.clc.uc.edu/fankhauser/Labs/Microbiology/Growth_Curve/Growth_Curve.htm