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Centre number	Candidate number
Surname	
Forename(s)	
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A-level FURTHER MATHEMATICS

Paper 2

Exam Date

Morning

Time allowed: 2 hours

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet. You do not necessarily need to use all the space provided.

Answer all questions in the spaces provided.

1 Given that $z_1 = 4e^{i\frac{\pi}{3}}$ and $z_2 = 2e^{i\frac{\pi}{4}}$ state the value of $arg\left(\frac{z_1}{z_2}\right)$

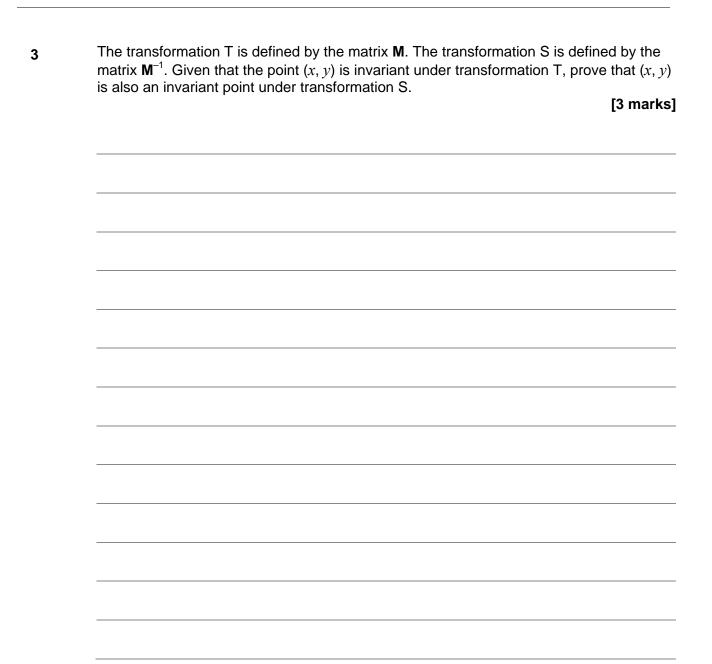
Circle your answer.

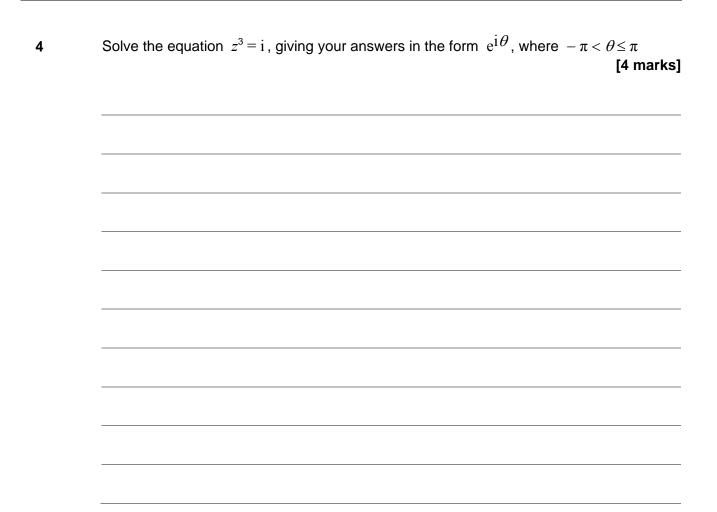
[1 mark]

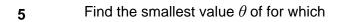
π	4	7π	2
12	3	12	2

2

[3 marks]







$$(\cos \theta + i \sin \theta)^5 = \frac{1}{\sqrt{2}}(1-i) \left\{ \theta \in \mathbb{R} : \theta > 0 \right\}$$
[4 marks]

6	Prove that $8^n - 7n + 6$ is divisible by 7 for all integers $n \ge 0$	[5 marks]

7 A small, hollow, plastic ball, of mass *m* kg is at rest on a point O on a polished horizontal surface. The ball is attached to two identical springs. The other ends of the springs are attached to the points P and Q which are 1.8 metres apart on a straight line through O.

The ball is struck so that it moves away from O, towards P with a speed of 0.75 m s⁻¹.

As the ball moves, its displacement from O is *x* metres at time *t* seconds after the motion starts.

The force that each of the springs applies to the ball is 12.5mx newtons towards O.

The ball is to be modelled as a particle. The surface is assumed to be smooth and it is assumed that the forces applied to the ball by the springs are the only horizontal forces acting on the ball.

7 (a) Find the minimum distance of the ball from P, in the subsequent motion.

[5 marks]

		[2 marks]	
	Explain your answer with reference to the model.		
	Is the minimum distance predicted by the model likely to be too big or too small	?	
7 (b)	In practice the minimum distance predicted by the model is incorrect.		

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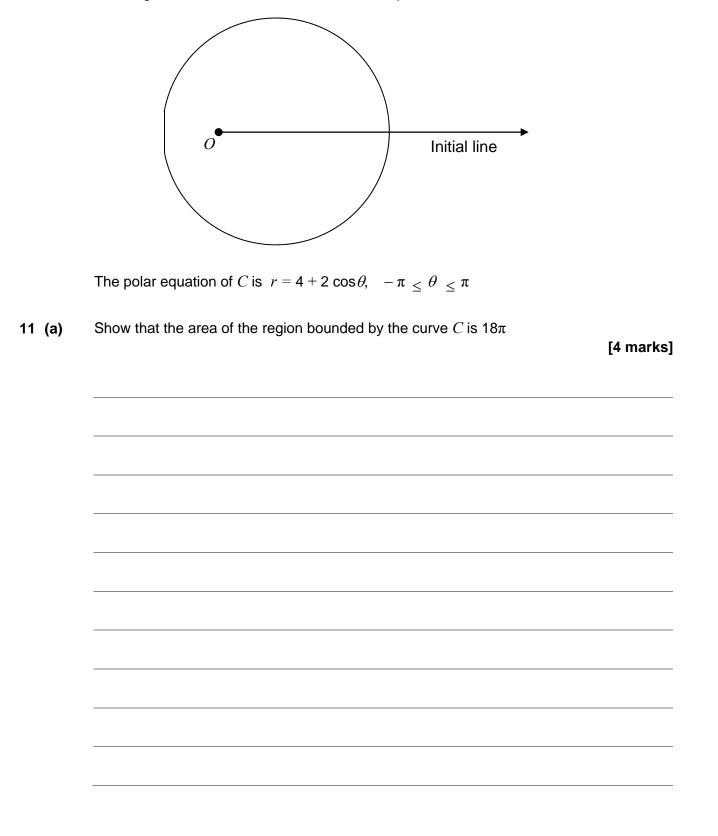
A student claims: 9 "Given any two non-zero square matrices, **A** and **B**, then $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ " 9 (a) Explain why the student's claim is incorrect giving a counter example. [2 marks] 9 (b) Refine the student's claim to make it fully correct. [1 mark]

9 (c) Prove that your answer to part (b) is correct. [3 marks]

10 Evaluate the improper integral $\int_0^\infty \frac{4x-30}{(x^2+5)(3x+2)} dx$, showing the limiting process used.

Give your answer as a single term.

[8 marks]



11 The diagram shows a sketch of a curve *C*, the pole *O* and the initial line.

Points *A* and *B* lie on the curve *C* such that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and *AOB* is an equilateral 11 (b) triangle. Find the polar equation of the line segment AB[4 marks]

12 (b) Given that MU = UD, where D is a diagonal matrix, find possible matrices for D and U. [8 marks]

13 S is a singular matrix such that

det **S** =
$$\begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix}$$

Express the possible values of x in terms of a and b.

[7 marks]

14 Given that the vectors **a** and **b** are perpendicular, prove that

 $|(\mathbf{a} + 5\mathbf{b}) \times (\mathbf{a} - 4\mathbf{b})| = k|\mathbf{a}||\mathbf{b}|$, where k is an integer to be found.

Explicitly state any properties of the vector product that you use within your proof. [9 marks]



15 (c) Hence show that
$$\sum_{n=1}^{\infty} \frac{1}{4^{n-1}} \cos 2n\theta = \frac{16\cos 2\theta - 4}{17 - 8\cos 2\theta}$$
[4 marks]

[4 marks]

[4 marks]

[4 marks]

[4 marks]

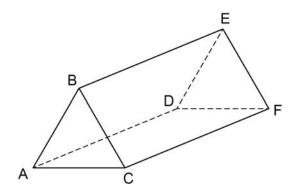
[1 mark]

[1 mark]

[1 mark]

21

16 A designer is using a computer aided design system to design part of a building. He models part of a roof as a triangular prism ABCDEF with parallel triangular ends ABC and DEF, and a rectangular base ACFD. He uses the metre as the unit of length.



The coordinates of B, C and D are (3, 1, 11), (9, 3, 4) and (-4, 12, 4) respectively.

He uses the equation x - 3y = 0 for the plane ABC.

He uses
$$\begin{bmatrix} \mathbf{r} - \begin{pmatrix} -4\\12\\4 \end{bmatrix} \times \begin{pmatrix} 4\\-12\\0 \end{pmatrix} = \begin{pmatrix} 0\\0\\0 \end{pmatrix}$$
 for the equation of the line AD.

Find the volume of the space enclosed inside this section of the roof.

[9 marks]



END OF QUESTIONS

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