

General Certificate of Education
Advanced Level Examination

MATHEMATICS A

Statistics 2

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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S2 Paper A – Marking Guide

- | | | | | |
|-------|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|-----|
| 1. | (a) | $P(X > 5) = 1 - F(5) = 1 - \frac{1}{64}(80 - 25) = \frac{9}{64}$ | M1 A1 | |
| | (b) | $f(x) = F'(x) = \frac{1}{64}(16 - 2x)$
$\therefore f(x) = \begin{cases} \frac{1}{32}(8 - x), & 0 \leq x \leq 8, \\ 0, & \text{otherwise.} \end{cases}$ | M1 A1 | |
| | | | A1 | (5) |
| <hr/> | | | | |
| 2. | (a) | let $P(X=0) = x$
$\therefore x + 0.8x + (0.8)^2x + (0.8)^3x + \dots = 1$
$\frac{x}{1-0.8} = 1 \quad \therefore x = 0.2$ | M1 A1 | |
| | (b) | geometric dist. [Geo(0.2)] | B1 | |
| | (c) | $E(X) = \frac{1}{0.2} = 5, \quad \text{Var}(X) = \frac{0.8}{0.2^2} = 20$ | B2 | (7) |
| <hr/> | | | | |
| 3. | (a) | let $X =$ no. out of 30 who visit advertiser's site $\therefore X \sim B(30, \frac{1}{40})$
$P(X \leq 1) = (\frac{39}{40})^{30} + 30(\frac{1}{40})(\frac{39}{40})^{29}$
$= 0.828$ (3sf) | B1
M1 A1
A1 | |
| | (b) | let $Y =$ no. out of 200 who visit advertiser's site $\therefore Y \sim B(200, \frac{1}{40})$
using Po approx. $Y \approx \sim \text{Po}(5)$
$P(Y > 10) = 1 - P(Y \leq 10)$
$\approx 1 - 0.9863 = 0.0137$ (3sf) | M1 A1
M1
A1 | (8) |
| <hr/> | | | | |
| 4. | (a) | let $F =$ time on French and $E =$ time on English
let $A = F + E \therefore A \sim N(55 + 90, 10^2 + 18^2) = \sim N(145, 424)$
$P(A > 120) = P(Z > \frac{120-145}{\sqrt{424}})$
$= P(Z > -1.21) = 0.88686 = 0.887$ (3sf) | M1 A1
M1
A1 | |
| | (b) | $P(E > 2F) = P(E - 2F > 0)$
let $B = E - 2F \therefore B \sim N(90 - 2 \times 55, 18^2 + 4 \times 10^2) = \sim N(-20, 724)$
$P(B > 0) = P(Z > \frac{0+20}{\sqrt{724}}) = P(Z > 0.74) = 1 - 0.77035 = 0.230$ (3sf) | M1
M1 A1
M1 A1 | (9) |
| <hr/> | | | | |

5.	expected freq. males/watched = $\frac{36 \times 40}{80} = 18$, males/stranded = $\frac{16 \times 40}{80} = 8$	M1 A1																												
	giving expected freqs																													
	18 8 14																													
	18 8 14	A1																												
	H_0 : no difference in preference of males and females																													
	H_1 : difference in preference of males and females	B1																												
	<table border="0"> <thead> <tr> <th>O</th> <th>E</th> <th>$(O - E)$</th> <th>$\frac{(O - E)^2}{E}$</th> </tr> </thead> <tbody> <tr> <td>21</td> <td>18</td> <td>3</td> <td>0.5</td> </tr> <tr> <td>6</td> <td>8</td> <td>-2</td> <td>0.5</td> </tr> <tr> <td>13</td> <td>14</td> <td>-1</td> <td>0.0714</td> </tr> <tr> <td>15</td> <td>18</td> <td>-3</td> <td>0.5</td> </tr> <tr> <td>10</td> <td>8</td> <td>2</td> <td>0.5</td> </tr> <tr> <td>15</td> <td>14</td> <td>1</td> <td>0.0714</td> </tr> </tbody> </table>	O	E	$(O - E)$	$\frac{(O - E)^2}{E}$	21	18	3	0.5	6	8	-2	0.5	13	14	-1	0.0714	15	18	-3	0.5	10	8	2	0.5	15	14	1	0.0714	
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	$\therefore \sum \frac{(O - E)^2}{E} = 2.143$	M1 A2																												
	$\nu = 2, \chi^2_{\text{crit}}(10\%) = 4.605$	M1 A1																												
	$2.143 < 4.605 \therefore$ not significant																													
	there is no evidence of a difference in preference of males and females	A1 (10)																												

6.	(a) Poisson with $\lambda = 4$	B1
	(b) e.g. more people shopping \therefore probably sell more so λ higher	B1
	(c) (i) let $X =$ no. of sales per hour $\therefore X \sim \text{Po}(4)$	
	$P(X > 4) = 1 - P(X \leq 4) = 1 - 0.6288 = 0.371$ (3sf)	M1 A1
	(ii) let $Y =$ no. of sales per half-hour $\therefore Y \sim \text{Po}(2)$	M1
	$P(Y = 0) = 0.1353 = 0.135$ (3sf)	A1
	(d) $H_0 : \lambda = 4$ $H_1 : \lambda > 4$	B1
	$P(X \geq 9) = 1 - P(X \leq 8) = 1 - 0.9786 = 0.0214$	M1 A1
	less than 5% \therefore significant, evidence of increase	A1 (10)

7.	H_0 : B(16, 0.1) is a suitable model																					
	H_1 : B(16, 0.1) is not a suitable model	B1																				
	$P(0) = (0.9)^{16} = 0.1853$																					
	$P(1) = 16(0.1)(0.9)^{15} = 0.3294$																					
	$P(2) = \frac{16 \times 15}{2} (0.1)^2 (0.9)^{14} = 0.2745$																					
	$P(3) = \frac{16 \times 15 \times 14}{3 \times 2} (0.1)^3 (0.9)^{13} = 0.1423$	M1 A1																				
	$P(4) = \frac{16 \times 15 \times 14 \times 13}{4 \times 3 \times 2} (0.1)^4 (0.9)^{12} = 0.0514$																					
	$\times 50$ to give exp. freqs then freq of $\geq 5 = (50 - \text{sum of others})$	M1																				
	\therefore exp. freqs are 9.27, 16.47, 13.73, 7.12, 2.57, 0.84	A1																				
	combining groups ≥ 3	M1																				
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16	10.53	5.47	2.8415																			
	$\therefore \sum \frac{(O - E)^2}{E} = 8.379$	M1 A1																				
	$\nu = 4 - 1 = 3, \chi^2_{\text{crit}}(5\%) = 7.815$	M1 A1																				
	$8.379 > 7.815 \therefore$ reject H_0																					
	B(16, 0.1) is not a suitable model	A1 (11)																				

Total (60)

Performance Record – S2 Paper A

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	c.d.f., p.d.f.	geometric	binomial, Po approx.	linear comb. of Normal r.v.	conting. table	Poisson, hyp. test	goodness of fit, binomial	
Marks	5	7	8	9	10	10	11	60
Student								