Further Pure 1 Past Paper Questions Pack A

Taken from MAP1, MAP2, MAP3, MAP4, MAP6

Parabolas, Ellipses and Hyperbolas

Pure 3 June 2002

3 A curve has equation $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$.

(a) Find the y–coordinates of the two points on the curve at which the x–coordinate is 2.
(2 marks)

Rational Functions and Asymptotes

Pure 2 June 2001

5 (a) Sketch the graph of $y = \dfrac{2x - 1}{x + 1}$ where $x \neq -1$. Indicate the asymptotes and the coordinates of the points of intersection of the curve with the axes. *(4 marks)*

(b) Solve the inequality

$$\frac{2x - 1}{x + 1} < 5.$$

(4 marks)

Pure 2 June 2002

3 Sketch the graph of $y = \dfrac{x}{x - 2}$ where $x \neq 2$.

Indicate the asymptotes and state their equations. *(5 marks)*

Pure 2 June 2003

7 A curve C has the equation

$$y = \frac{2x + 1}{x + 2}, \qquad x \neq -2.$$

(a) Express the equation of C in the form

$$y = A + \frac{B}{x + 2},$$

where A and B are numbers to be found. *(3 marks)*

(b) Sketch the curve C. Indicate the asymptotes and the points of intersection of the curve with the axes.
(4 marks)

Complex Numbers / Roots of Quadratic Equations

Pure 4 June 2004

1 (a) Show that $(3 - \mathrm{i})^2 = 8 - 6\mathrm{i}$. *(1 mark)*

(b) The quadratic equation

$$az^2 + bz + 10\mathrm{i} = 0,$$

where a and b are real, has a root $3 - \mathrm{i}$.

(i) Show that $a = 3$ and find the value of b. *(6 marks)*

(ii) Determine the other root of the quadratic equation, giving your answer in the form $p + \mathrm{i}q$. *(3 marks)*

Pure 2 June 2001

2 The roots of the quadratic equation

$$x^2 - 5x + 3 = 0$$

are α and β. Form a quadratic equation whose roots are $\alpha + 1$ and $\beta + 1$, giving your answer in the form $x^2 + px + q = 0$, where p and q are integers to be determined. *(4 marks)*

Pure 2 June 2003

2 The quadratic equation

$$x^2 + px + 2 = 0$$

has roots α and β.

(a) Write down the value of $\alpha\beta$. *(1 mark)*

(b) Express in terms of p:

(i) $\alpha + \beta$; *(1 mark)*

(ii) $\alpha^2 + \beta^2$. *(2 marks)*

(c) Given that $\alpha^2 + \beta^2 = 5$, find the possible values of p. *(1 mark)*

Pure 2 Jan 2004

1 (a) The quadratic equation $2x^2 - 6x + 1 = 0$ has roots α and β.

Write down the numerical values of:

(i) $\alpha\beta$; *(1 mark)*

(ii) $\alpha + \beta$. *(1 mark)*

(b) Another quadratic equation has roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

Find the numerical values of:

(i) $\dfrac{1}{\alpha} \times \dfrac{1}{\beta}$; *(1 mark)*

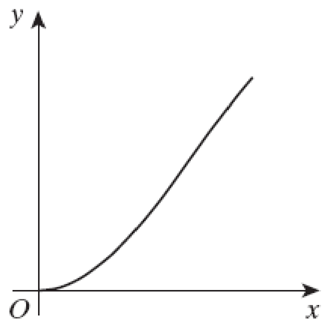
(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$. *(2 marks)*

(c) Hence, or otherwise, find the quadratic equation with roots $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$, writing your answer in the form $x^2 + px + q = 0$. *(2 marks)*

Numerical Methods

Pure 1 June 2001

3



The diagram shows the graph of

$$y = x \sin x, \qquad 0 \leqslant x \leqslant \frac{\pi}{2}.$$

(a) Show, using a suitable diagram, that the equation

$$x \sin x = \cos x$$

has exactly one root in the interval $0 \leqslant x \leqslant \frac{\pi}{2}$. *(2 marks)*

(b) Denoting this root by α, show that α is also a root of the equation $f(x) = 0$, where

$$f(x) = \tan x - \frac{1}{x}.$$ *(2 marks)*

(c) Show that $f(0.8) \approx -0.220$ and find the value of $f(0.9)$ to three decimal places. *(2 marks)*

(d) Use linear interpolation once to estimate the value of α, giving your answer to two decimal places. *(2 marks)*

Pure 1 Jan 2002

5 (a) (i) Show that the equation

$$\tan\theta - 2\theta = 0,$$

where the angle θ is given in **radians**, has a root between 1 and 1.2. *(2 marks)*

(ii) Use interval bisection to find an interval of width 0.05 within which the root must lie. *(2 marks)*

Pure 1 June 2002

1 (a) Show that the equation

$$x^4 = 5 - 2x$$

has a root between 1.2 and 1.3. *(3 marks)*

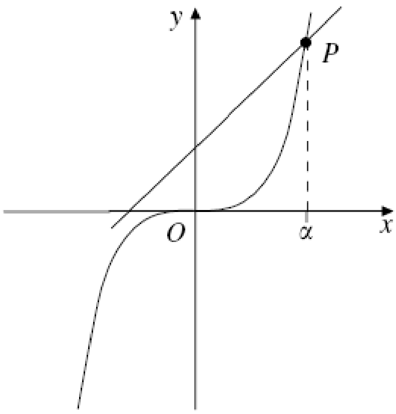
(b) Use the method of interval bisection to find whether this root is nearer to 1.2 or to 1.3. *(2 marks)*

Pure 1 Jan 2003

2 The diagram shows the graphs of

$$y = x^3 \quad \text{and} \quad y = x + 1,$$

intersecting at the point P, which has x-coordinate α.



(a) Show that, at P,

$$x^3 - x - 1 = 0\,. \qquad \textit{(1 mark)}$$

(b) (i) Show that α lies in the interval between 1.2 and 1.4. *(3 marks)*

(ii) Use interval bisection **twice**, starting with the interval in part (b)(i), to find an interval of width 0.05 within which α must lie. *(3 marks)*

(iii) Hence give the value of α to one decimal place. *(1 mark)*

Pure 2 June 2001

7 (a) Sketch, on the same diagram, the graphs of $y = \ln x$ and $y = \frac{3}{x}$ for $x > 0$. *(2 marks)*

(b) (i) Show that the equation $\ln x - \frac{3}{x} = 0$ has a root between $x = 2$ and $x = 3$. *(2 marks)*

(ii) With a starting value of 2.5, use the Newton-Raphson method once to find a second approximation to this root. *(4 marks)*

Pure 2 June 2003

4 (a) Show that the equation

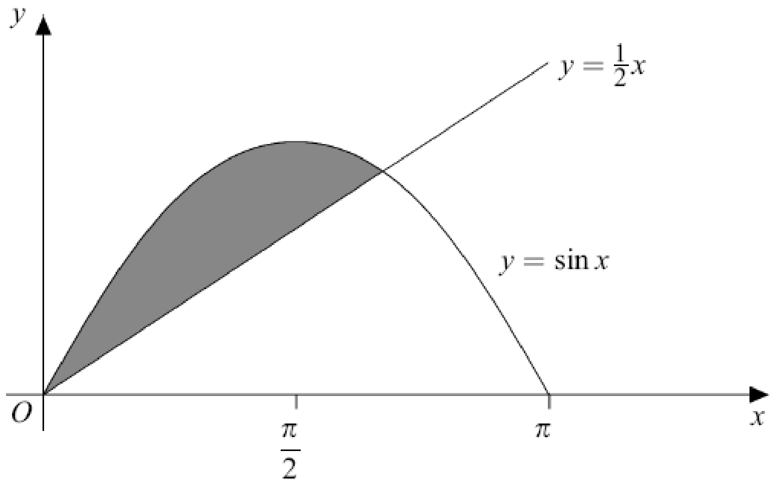
$$2\cos x - \frac{1}{x} = 0, \qquad 0 < x < \frac{\pi}{2},$$

has a root between $x = 0.6$ and $x = 0.7$. *(3 marks)*

(b) Taking 0.6 as a first approximation to the root, use the Newton-Raphson method once to find a second approximation. Give your answer to three decimal places. *(5 marks)*

Pure 2 Jan 2004

6 The diagram below shows the graphs of $y = \sin x$ and $y = \frac{1}{2}x$, for $0 \leqslant x \leqslant \pi$.



(a) Show that the equation $\sin x - \frac{1}{2}x = 0$ has a root in the interval $1 \leqslant x \leqslant 2$, where x is measured in radians. *(2 marks)*

(b) (i) Given that $f(x) = \sin x - \frac{1}{2}x$, find $f'(x)$. *(1 mark)*

(ii) Use a single application of the Newton–Raphson method, with an initial value $x = 2$, to show that the root of the equation $\sin x - \frac{1}{2}x = 0$ in the interval $1 \leqslant x \leqslant 2$ is approximately 1.9. *(3 marks)*

Pure 3 June 2001

5 A curve satisfies the differential equation $\frac{dy}{dx} = \sqrt{9 - x^2}$.

Starting at the point $(0, 3)$ on the curve, use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 1$, giving your answer to two decimal places. *(5 marks)*

Pure 3 Jan 2002

3 A curve satisfies the differential equation

$$\frac{dy}{dx} = \frac{x + y}{3 - y}.$$

(a) Starting at the point $(-2, 1)$ on the curve, use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = -1$, giving your answer to two decimal places. *(4 marks)*

(b) State a way in which the method in part (a) could be improved. *(1 mark)*

Pure 3 Jan 2003

4 A curve satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{x^2 - 5}.$$

Starting at the point $(3, 1)$ on the curve, use a step by step method with a step length of 0.5 to estimate the value of y at $x = 4$. Give your answer to two decimal places. *(5 marks)*

Pure 3 June 2003

6 (a) Given the differential equation

$$\frac{dx}{dt} = \frac{10 - x}{5},$$

obtain a numerical solution, starting at $x = 1$ and $t = 0$ and using a step length of 0.3, to show that x is approximately 2 when $t = 0.6$. *(4 marks)*

Pure 3 Jan 2004

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{1 + x^3}.$$

Starting at the point $(1, 0.5)$ on the curve, use a step-by-step method with a step length of 0.25 to estimate the value of y at $x = 1.5$, giving your answer to two decimal places. *(5 marks)*

Matrix Transformations

Pure 6 Jan 2002

4 A transformation T_1 is represented by the matrix

$$\mathbf{M}_1 = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}.$$

(a) Give a geometrical description of T_1. *(3 marks)*

The transformation T_2 is a reflection in the line $y = \sqrt{3}x$.

(b) Find the matrix $\mathbf{M}_2$ which represents the transformation T_2. *(3 marks)*

(c) (i) Find the matrix representing the transformation T_2 followed by T_1. *(2 marks)*

(ii) Give a geometrical description of this combined transformation. *(3 marks)*

Pure 6 Jan 2003

1 (a) Find the 2×2 matrix which represents, in two dimensions, a *clockwise* rotation through an angle of θ about the origin. *(2 marks)*

(b) Find the matrix which transforms

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ to } \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 \\ 2 \end{bmatrix} \text{ to } \begin{bmatrix} 6 \\ 6 \end{bmatrix}.$$ *(3 marks)*

Pure 6 June 2003

2 The transformation T is represented by the matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}.$$

(a) Give a geometrical description of T. *(3 marks)*

(b) Find the smallest positive value of n for which

$$\mathbf{M}^n = \mathbf{I}.$$ *(2 marks)*