Pure Core 1 Past Paper Questions Pack B

Taken from MAP2

June 2001

3 (a) Solve the simultaneous equations

$$y = x + 1,$$

$$x^2 - 8x + y^2 - 2y + 9 = 0.$$ *(4 marks)*

(b) Hence describe the geometrical relationship between the straight line with equation $y = x + 1$, and the circle with equation $x^2 - 8x + y^2 - 2y + 9 = 0$, giving a reason for your answer. *(2 marks)*

4 (a) Prove that, if the polynomial $f(x)$ has a factor $(x - a)$, then $f(a) = 0$. *(2 marks)*

(b) The polynominal $f(x) = x^3 + px^2 + qx + 6$ has a factor $(x - 1)$.
When $f(x)$ is divided by $x + 1$, there is a remainder of 8.
Find the values of p and q. *(4 marks)*

5 (a) Sketch the graph of $y = \frac{2x - 1}{x + 1}$ where $x \neq -1$. Indicate the asymptotes and the coordinates of the points of intersection of the curve with the axes. *(4 marks)*

(b) Solve the inequality

$$\frac{2x - 1}{x + 1} < 5.$$ *(4 marks)*

January 2002

6 The line joining the points $A\ (0, 5)$ and $B\ (4, 1)$ is a tangent to a circle whose centre, C, is at the point $(5, 4)$.

(a) Find the equation of the line AB. *(2 marks)*

(b) Find the equation of the line through C which is perpendicular to AB. *(2 marks)*

(c) Find the coordinates of the point of contact of the line AB with the circle. *(2 marks)*

(d) Find the equation of the circle. *(2 marks)*

June 2002

1 Divide $x^3 + 2x^2 - 5x - 6$ by $x + 1$. *(3 marks)*

6 A circle has equation $x^2 + y^2 + 2x - 6y = 0$.

(a) Find the radius of the circle, and the coordinates of its centre. *(4 marks)*

(b) Find the equation of the tangent to the circle at the point $(2, 4)$. *(5 marks)*

January 2003

1 The polynomial $f(x)$ is given by

$$f(x) = x^3 + px^2 + x + 54,$$

where p is a real number. When $f(x)$ is divided by $x + 3$, the remainder is -3.

Use the Remainder Theorem to find the value of p. *(3 marks)*

3 A circle has the equation

$$(x - 3)^2 + (y - 4)^2 = 16.$$

The point A has coordinates $\left(\frac{3}{5}, \frac{4}{5}\right)$.

(a) Show that A lies on the circle. *(1 mark)*

(b) Sketch the circle. *(2 marks)*

(c) Show that the normal to the circle at A passes through the origin. *(3 marks)*

(d) Find the equation of the tangent to the circle at A, giving your answer in the form

$$ax + by = c,$$

where a, b and c are integers. *(4 marks)*

June 2003

6 A circle has the equation

$$x^2 + y^2 + 4x - 14y + 4 = 0.$$

(a) Find the radius of the circle and the coordinates of its centre. *(5 marks)*

(b) Sketch the circle. *(2 marks)*

(c) Find the length of a tangent from the point $P(6, 8)$ to the circle. *(4 marks)*

January 2004

2 A circle has equation

$$x^2 + y^2 - 4x + 4y - 12 = 0.$$

(a) Find:

(i) the coordinates of the centre of the circle;

(ii) the radius of the circle. *(5 marks)*

(b) Find the coordinates of the **two** points where the circle crosses the x-axis. *(3 marks)*

(c) Find the equation of the tangent to the circle at the point $(4, 2)$. *(4 marks)*

June 2004

6 (a) The circle $(x-4)^2+(y-3)^2=4$ has centre C and radius r.

Write down:

(i) the coordinates of C;

(ii) the value of r. *(2 marks)*

(b) The line $y=x+1$ intersects this circle at two points A and B.

(i) Find the coordinates of A and B. *(5 marks)*