GCE Examinations
Advanced / Advanced Subsidiary

Core Mathematics C1

Paper 3

Time: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**

1. Write down the exact value of $8^{-\frac{2}{3}}$. [2]

2. Express $\left(2\sqrt{3} + 3\sqrt{2}\right)^2$ in the form $a + b\sqrt{c}$, stating the values of the integers a, b and c. [3]

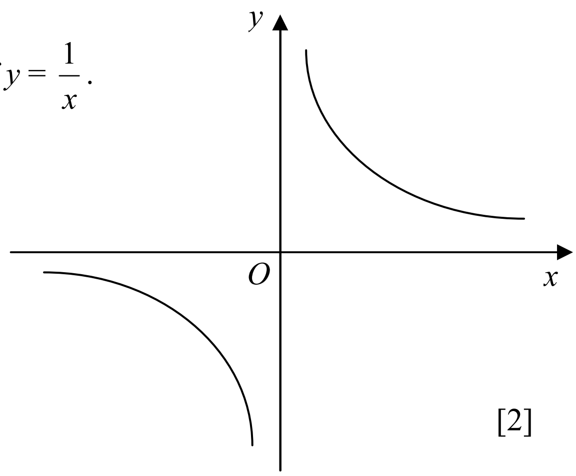
3. Given that $f(x) = x + \sqrt{x}$, find expressions for $f'(x)$ and $f''(x)$. [4]

4. The straight line l has equation $2x - 5y = 7$ and the point A has coordinates $(3, -2)$.

   i) Find the gradient of l. [1]

   ii) Find the equation of the straight line that passes through A and is perpendicular to l. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [3]

5. The diagram shows a sketch of the graph of $y = \frac{1}{x}$.



   i) Sketch the graph of $y = \frac{1}{x^2}$. [2]

   ii) On the same diagram, sketch the graph of $y = \frac{1}{(x+2)^2}$. [2]

   iii) State the number of solutions of the equation $\frac{1}{x^2} = \frac{1}{(x+2)^2}$. [1]

   iv) Solve the equation $\frac{1}{x^2} = \frac{1}{(x+2)^2}$. [2]

6. Find the set of values of x for which the function given by $f(x) = 4x^3 + 21x^2 - 24x + 1$ is increasing. [4]

   {Hint: a function is increasing if it's gradient is positive, so work out $f'(x)$ and solve the inequality $f'(x) > 0$.}

7. The circle C has equation $x^2 + y^2 - 6x - 12y + 28 = 0$.

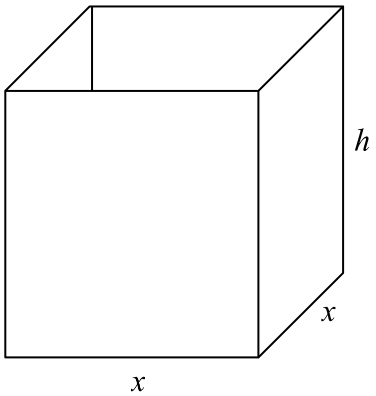
a) Find the co-ordinates of the centre of C. [2]

The line $y = x - 2$ intersects C at the points A and B.

b) Find the length AB in the form $k\sqrt{2}$. [4]
{Hint: solve simultaneously.}

8. Find the set of values of a for which the equation $ax^2 - 6x + a = 0$ has two distinct real roots. **{Hint: discriminant!}** [4]

9.



The diagram shows an open rectangular tank, of height h metres, with a horizontal square base of side x metres. The tank can hold a volume of 13.5 m^3 of water and the internal surface area of the tank is S m^2.

i) Show that $$S = x^2 + \frac{54}{x}.$$ [3]

ii) Differentiate S with respect to x and hence find the dimensions of the tank when S is a minimum. Show clearly that, in this case, S is a minimum and not a maximum. [6]

ANSWERS.

1. $\frac{1}{4}$.

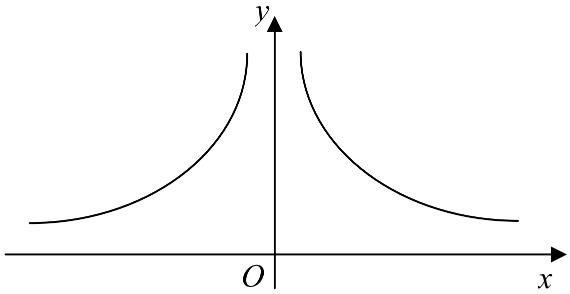
2. $30 + 12\sqrt{6}$, $a = 30$, $b = 12$, $c = 6$.

3. $f'(x) = 1 + \frac{1}{2}x^{-\frac{1}{2}}$ or $1 + \frac{1}{2\sqrt{x}}$.

$f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$.

4. i) $\frac{2}{5}$ ii) $5x + 2y - 11 = 0$.

5. i)



ii) {The graph of $y = \frac{1}{(x+2)^2}$ is a translation of the graph of $y = \frac{1}{x^2}$ of 2 units along the negative x-axis.}

iii) The solutions to the equations are the points where the graphs $y = \frac{1}{x^2}$ and $y = \frac{1}{(x+2)^2}$ intersect and so there is only one point of intersection.

iv) $x = -1$.

6. $x \leq -4$ or $x \geq \frac{1}{2}$.

7. a) (3, 6) b) $A = (4, 2)$, $B = (7, 5)$. $AB = 3\sqrt{2}$.

8. $-3 < a < 3$.

9. ii) $\frac{dS}{dx} = 2x - \frac{54}{x^2}$.

S is a minimum when $x = 3$ and $h = 1.5$.