ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics – Pure Core

SPECIMEN UNITS AND MARK SCHEMES

ADVANCED SUBSIDIARY MATHEMATICS (5361)
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

ADVANCED MATHEMATICS (6361)
ADVANCED PURE MATHEMATICS (6366)
ADVANCED FURTHER MATHEMATICS (6371)

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

In addition to this paper you will require: • an 8-page answer book; • the AQA booklet of formulae and statistical tables. You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Calculators (scientific and graphical) are **not** permitted in this paper.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The line L has equation

$$x + y = 9$$

and the curve C has equation

$$y = x^2 + 3$$

(a) Sketch on one pair of axes the line L and the curve C. Indicate the coordinates of their points of intersection with the axes. *(3 marks)*

(b) Show that the x-coordinates of the points of intersection of L and C satisfy the equation

$$x^2 + x - 6 = 0$$

(2 marks)

(c) Hence calculate the coordinates of the points of intersection of L and C. *(4 marks)*

2 The line AB has equation $5x - 2y = 7$.

The point A has coordinates $(1, -1)$ and the point B has coordinates $(3, k)$.

(a) (i) Find the value of k. *(1 mark)*

(ii) Find the gradient of AB. *(2 marks)*

(b) The point C has coordinates $(-6, -2)$. Show that AC has length $p\sqrt{2}$, where p is an integer. *(3 marks)*

3 (a) Given that $(x - 1)$ is a factor of $f(x)$, where

$$f(x) = x^3 - 4x^2 - kx + 10$$

show that $k = 7$. *(2 marks)*

(b) Divide $f(x)$ by $(x - 1)$ to find a quadratic factor of $f(x)$. *(2 marks)*

(c) Write $f(x)$ as a product of three linear factors. *(2 marks)*

(d) Calculate the remainder when $f(x)$ is divided by $(x - 2)$. *(2 marks)*

4 The number x satisifies the equation

$$x^2 + mx + 16 = 0$$

where m is a constant.

Find the values of m for which this equation has:

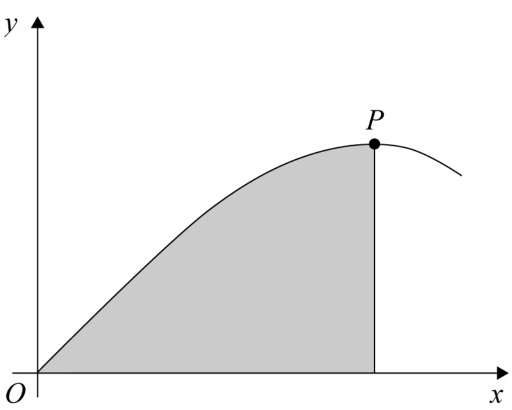
(a) equal roots; *(2 marks)*

(b) two distinct real roots; *(2 marks)*

(c) no real roots. *(2 marks)*

5 The diagram shows a part of the graph of

$$y = x - 2x^4$$



(a) (i) Find $\frac{dy}{dx}$. *(2 marks)*

(ii) Show that the x-coordinate of the stationary point P is $\frac{1}{2}$. *(3 marks)*

(iii) Find the y-coordinate of P. *(1 mark)*

(b) Find the area of the shaded region. *(5 marks)*

Turn over ►

6 A circle C has equation

$$x^2 + y^2 - 10x = 0$$

(a) By completing the square, express this equation in the form

$$(x - a)^2 + y^2 = r^2$$

(3 marks)

(b) Write down the radius and the coordinates of the centre of the circle C. *(2 marks)*

(c) Describe a geometrical transformation by which C can be obtained from the circle with equation

$$x^2 + y^2 = r^2$$

(2 marks)

(d) The point P, which has coordinates (9, 3), lies on the circle C.

(i) Show that the line which passes through P and the centre of C has gradient $\frac{3}{4}$ *(2 marks)*

(ii) Find the equation of the tangent to the circle C at the point P. Give your answer in the form $y = mx + c$. *(4 marks)*

7 (a) Express $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ in the form $a\sqrt{2}+b$, where a and b are integers. *(4 marks)*

(b) Solve the inequality

$$\sqrt{2}\left(x-\sqrt{2}\right) < x + 2\sqrt{2}$$

(4 marks)

8 A curve has equation

$$y = x^4 - 8x^3 + 16x^2 + 8$$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. *(5 marks)*

(b) Find the three values of x for which $\frac{dy}{dx} = 0$. *(4 marks)*

(c) Determine the coordinates of the point at which y has a maximum value. *(5 marks)*

END OF QUESTIONS

MPC1 Specimen

Question	Solution	Marks	Total	Comments
1(a)	Sketches of L and C Coordinates (0, 9), (9, 0) indicated Coordinates (0, 3) indicated	M1 A1 A1	 3	General shape Accept labels on sketch ditto
(b)	Equating expressions for y $x^2 + x - 6 = 0$	M1 A1	 2	oe convincingly shown (**ag**)
(c)	Solving quadratic $x = -3$ or $x = 2$ Points are (−3, 12) and (2, 7)	M1 A1 A1A1	 4	Two solutions needed A1 if not clearly paired
	Total		**9**	
2(a)(i)	$k = 4$	B1	1	
(ii)	Gradient = $\dfrac{4-(-1)}{3-1}$ $= \frac{5}{2}$	M1 A1√	 2	or use of equation of AB ft wrong value of k
(b)	Distance formula $AC = \sqrt{50}$ $= 5\sqrt{2}$	M1 A1 A1	 3	stated or used
	Total		**6**	
3(a)	Use of factor theorem $1 - 4 - k + 10 = 0$, so $k = 7$	M1 A1	 2	or complete division convincingly shown (**ag**)
(b)	Quotient is $x^2 - 3x - 10$	B2	2	B1 if $-3x$ or -10 correct
(c)	$f(x) = (x - 1)(x + 2)(x - 5)$	B2	2	B1 if signs wrong
(d)	$f(2) = -12$ so remainder is -12	B1 B1√	 2	 ft wrong value for f(2)
	Total		**8**	
4(a)	$m^2 - 64 = 0$ $m = \pm 8$	M1 A1	 2	
(b)	$m^2 - 64 > 0$ $m < -8$ or $m > 8$	M1 A1	 2	
(c)	$m^2 - 64 < 0$ $-8 < m < 8$	M1 A1	 2	
	Total		**6**	

MPC1 (cont)

Question	Solution	Marks	Total	Comments
5(a)(i)	$y' = 1-8x^3$	M1A1	2	M1 if at least one term correct
(ii)	SP $\Rightarrow y' = 0$	M1		PI
	$\Rightarrow x^3 = \frac{1}{8}$	A1		
	$\Rightarrow x = \frac{1}{2}$ convincingly shown	A1	3	**ag**; 2/3 for verification
(iii)	$y_P = \frac{3}{8}$	B1	1	
(b)	$\int y\,dx = \frac{1}{2}x^2 - \frac{2}{5}x^5 \quad (+c)$	M1A1		M1 if at least one term correct
	Substitution of $x = \frac{1}{2}$	m1		
	Area = $\frac{1}{8} - \frac{1}{80}$	A1		First A1 awarded if at least one term
	$= \frac{9}{80}$	A1	5	correct
	Total		**11**	
6(a)	Use of $(x-5)^2 = x^2 - 10x + 25$	M1		
	$a = 5, r = 5$	A1A1	3	Condone RHS = 25
(b)	Radius 5	B1√		ft wrong value for *a*
	Centre (5, 0)	B1√	2	ditto
(c)	Translation	M1		Condone 'transformation' if clarified
	5 units in positive *x* direction	A1√	2	ft wrong value for *a*
(d)(i)	Use of formula for gradient	M1		
	Grad $\frac{3}{4}$ convincingly shown	A1	2	**ag**
(ii)	Grad of tangent is $-\frac{4}{3}$	B1		
	Tangent is $y-3 = -\frac{4}{3}(x-9)$	M1		
		A1√		ft wrong gradient
	i.e. $y = -\frac{4}{3}x + 15$	A1√	4	ft wrong gradient
	Total		**12**	
7(a)	Rationalising denominator	M1		
	Numerator becomes $2\sqrt{2}+3$	m1A1		
	Denom = 1, so ans is $2\sqrt{2}+3$	A1√	4	ft one small error in numerator
(b)	LHS = $\sqrt{2}x - 2$	B1		Allow $\sqrt{2}x - \sqrt{4}$
	$\sqrt{2}x - x < 2\sqrt{2} + 2$	M1		for isolating *x* terms
	Reasonable attempt at division	m1		
	$x < \dfrac{2\sqrt{2}+2}{\sqrt{2}-1}$	A1√	4	ft error in expanding LHS
	Total		**8**	

Question	Solution	Marks	Total	Comments
8(a)	$y' = 4x^3 - 24x^2 + 32x$	M1		at least one term correct
		A2		A1 with one error
	$y'' = 12x^2 - 48x + 32$	m1		at least one term correct
		A1√	5	ft numerical error in y'
(b)	$y' = 0$ if $x = 0$	B1		
	or if $x^2 - 6x + 8 = 0$	M1		
	ie $x = 2$ or $x = 4$	A2	4	A1 if only one small error
(c)	$y'' = 32, -16, 32$	M1		M1 if at least one correct
		A1√		ft numerical error in y''
	Relationship between sign of y'' and maximum/minimum	M1		
	Max at $x = 2$	A1		
	$y = 24$	A1	5	
	Total		**15**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

In addition to this paper you will require:
• an 8-page answer book.
• the AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

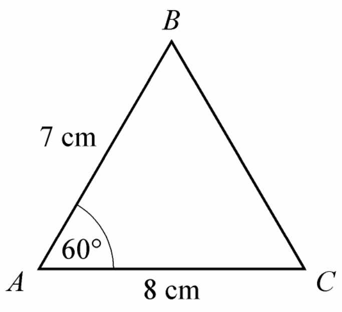
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

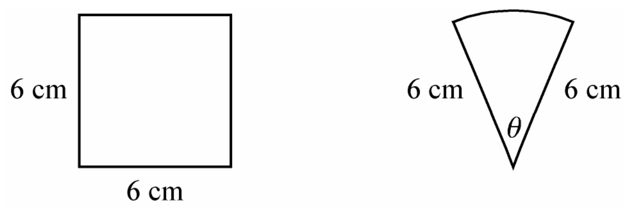
Answer **all** questions.

1 The diagram shows triangle *ABC*.



The lengths of *AB* and *AC* are 7cm and 8cm respectively. The size of angle *BAC* is 60°. Calculate the length of *BC*, giving your answer to 3 significant figures. *(3 marks)*

2 The diagrams show a square of side 6 cm and a sector of a circle of radius 6 cm and angle θ radians.



The area of the square is three times the area of the sector.

(a) Show that $\theta = \frac{2}{3}$. *(2 marks)*

(b) Show that the perimeter of the square is $1\frac{1}{2}$ times the perimeter of the sector. *(3 marks)*

3 The *n*th term of an arithmetic sequence is u_n, where

$$u_n = 10 + 0.5n$$

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Write down the common difference of the arithmetic sequence. *(1 mark)*

(c) Find the value of n for which $u_n = 25$. *(2 marks)*

(d) Evaluate $\sum_{n=1}^{30} u_n$. *(3 marks)*

4 (a) Given that

$$\log_a x = \log_a 5 + 2\log_a 3$$

where a is a positive constant, show that $x = 45$. *(3 marks)*

(b) (i) Write down the value of $\log_2 2$. *(1 mark)*

(ii) Given that

$$\log_2 y = \log_4 2$$

find the value of y. *(2 marks)*

5 The curve C is defined by the equation

$$y = 2x^2\sqrt{x} + \frac{1}{x^4} \quad \text{for } x > 0$$

(a) Write $x^2\sqrt{x}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(c) Find an equation of the tangent to the curve C at the point on the curve where $x = 1$. *(4 marks)*

(d) (i) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(2 marks)*

(ii) Hence deduce that the curve C has no maximum points. *(2 marks)*

6 The amount of money which Pauline pays into an insurance scheme is recorded each year. The amount which Pauline pays in during the nth year is £A_n. The first three values of A_n are given by:

$$A_1 = 800 \qquad A_2 = 650 \qquad A_3 = 530$$

The recorded amounts may be modelled by a law of the form

$$A_{n+1} = pA_n + q$$

where p and q are constants.

(a) Find the value of p and the value of q. *(5 marks)*

(b) Given that the amounts converge to a limiting value, £V, find an equation for V and hence find the value of V. *(3 marks)*

Turn over ►

7 (a) Express $\dfrac{x^5+1}{x^2}$ in the form $x^p + x^q$, where p and q are integers. *(2 marks)*

(b) Hence find the exact value of $\displaystyle\int_1^{\frac{3}{2}} \left(\frac{x^5+1}{x^2}\right) \mathrm{d}x$. *(5 marks)*

8 The angle θ radians, where $0 \le \theta \le 2\pi$, satisfies the equation

$$3 \tan \theta = 2 \cos \theta$$

(a) Show that $3 \sin \theta = 2 \cos^2 \theta$ *(1 mark)*

(b) Hence use an appropriate identity to show that

$$2 \sin^2 \theta + 3 \sin \theta - 2 = 0$$ *(3 marks)*

(c) (i) Solve the quadratic equation in part (b). Hence explain why the only possible value of $\sin \theta$ which will satisfy it is $\frac{1}{2}$. *(3 marks)*

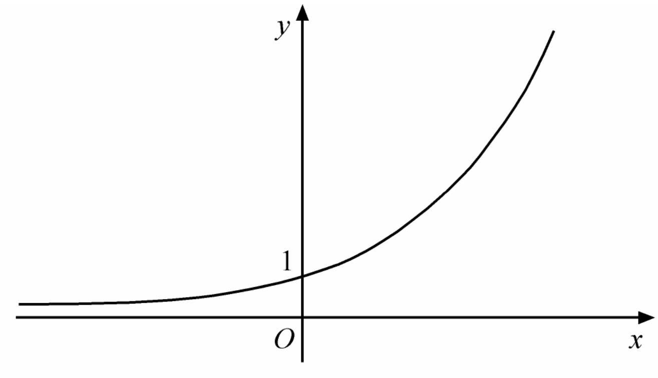
(ii) Find the values of θ for which $\sin\theta = \frac{1}{2}$ and $0 \leqslant \theta \leqslant 2\pi$. *(2 marks)*

(d) Hence write down the solutions of the equation

$$3 \tan 2x = 2 \cos 2x$$

that lie in the interval $0° \leqslant x \leqslant 180°$. *(3 marks)*

9 The diagram shows a sketch of the curve with equation $y = 4^x$.



(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximation for $\int_0^2 4^x \, dx$. *(4 marks)*

(ii) By considering the graph of $y = 4^x$, explain with the aid of a diagram whether your approximation will be an overestimate or an underestimate of the true value for $\int_0^2 4^x \, dx$. *(2 marks)*

(b) Describe the single transformation by which the curve with equation $y = 5 \times 4^x$ can be obtained from the curve with equation $y = 4^x$. *(2 marks)*

(c) Sketch the curve with equation $y = 4^{-x}$. *(1 mark)*

(d) The two curves $y = 5 \times 4^x$ and $y = 4^{-x}$ intersect at the point P.

(i) Show that the x-coordinate of the point P is a root of the equation $4^{2x} = 0.2$ *(2 marks)*

(ii) Solve this equation to find the x-coordinate of the point P. Give your answer to 5 significant figures. *(3 marks)*

END OF QUESTIONS

MPC2 Specimen

Question	Solution	Marks	Total	Comments
1	$BC^2 = 7^2 + 8^2 - 2\times7\times8\times\cos60^\circ$ $= 49 + 64 - 56$ $= 57 \Rightarrow BC = \sqrt{57} = 7.55$ to 3sf	M1 m1 A1	 3	
	Total		**3**	
2(a)	{Area of square} $= 3 \times \frac{1}{2}6^2\theta$ $6^2 = \frac{3}{2}6^2\theta \;\Rightarrow \theta = \frac{2}{3}$	M1 A1	 2	Use of $\frac{1}{2}r^2\theta$ PI **ag**
(b)	Arc length $= 6\theta$ Perimeter of sector $= (12 + 6\theta)$ $= 1\frac{1}{2}\left(12 + 6\times\frac{2}{3}\right) = 24$ = perimeter of square	B1 M1 A1	 3	 **ag**
	Total		**5**	
3(a)	$u_1 = 10.5$; $u_2 = 11$	B1B1	2	sc B1 for 10, 10.5
(b)	Common difference is 0.5	B1	1	
(c)	$10 + 0.5n = 25 \Rightarrow 0.5n = 25 - 10$ $\Rightarrow n = 30$	M1 A1	 2	
(d)	$\sum_{n=1}^{30} u_n$ = sum of AP with $n = 30$ $= \frac{30}{2}(10.5 + 25)$ $= 532.5$	M1 m1 A1	 3	 oe
	Total		**8**	
4(a)	$\log_a x = \log_a 5 + \log_a 3^2$ $\log_a x = \log_a[5 \times 3^2]$ $\Rightarrow x = 45$	M1 m1 A1	 3	PI **ag** convincingly found
(b)(i)	$\log_2 2 = 1$	B1	1	
(ii)	$\{\log_2 y =\}\ \log_4 2 = 0.5$ $\Rightarrow y = 2^{\frac{1}{2}} = \sqrt{2}$	B1 B1	 2	
	Total		**6**	

Question	Solution	Marks	Total	Comments
5(a)	$x^2\sqrt{x} = x^{\frac{5}{2}}$	B1	1	Accept $k = 2.5$
(b)	$y = 2x^{\frac{5}{2}} + x^{-4}$			
	$\frac{dy}{dx} = 5x^{\frac{3}{2}} - \frac{4}{x^5}$	M1 A1A1	3	One correct index ft A1 for each correct term
(c)	When $x = 1$, $y = 2+1 = 3$ When $x = 1$, $y' = 5-4 = 1$ Eqn. of tangent: $y - 3 = 1(x-1)$	B1 M1 m1 A1	4	Accept any valid form
(d)(i)	$\frac{d^2y}{dx^2} = \frac{15}{2}x^{\frac{1}{2}} + \frac{20}{x^6}$	B2,1√	2	ft each term provided equivalent demands ie indices one fractional one 'negative'
(ii)	Since $x>0$, $y''(x)$ is >0 so any turning point must be a minimum ie C has no maximum points	E2,1,0	2	E1 for attempt to find the sign of $y''(x)$
	Total		**12**	
6(a)	$650 = 800p + q$ $530 = 650p + q$	M1 A1 m1		For either equation Need both Full valid method to solve simultaneous equations
	$p = \frac{4}{5}$; $q = 10$	A1A1	5	
(b)	$V = pV + q$	M1		
	$V = \frac{q}{1-p}$	m1		
	$V = 50$	A1√	3	ft on 1 numerical slip in (a)
	Total		**8**	
7(a)	$x^3 + x^{-2}$	M1 A1	2	One power correct
(b)	$\frac{x^4}{4} - x^{-1}$	M1 A1√ A1		Index raised by 1 (either term) One term correct ft p, q. All correct
	$\left[\frac{\left(\frac{3}{2}\right)^4}{4} - \frac{2}{3}\right] - \left[\frac{1}{4} - 1\right]$	m1		Use of limits
	$= 1\frac{67}{192}$	A1	5	Must be an **exact** value
	Total		**7**	

MPC2 (cont)

Question	Solution	Marks	Total	Comments
8(a)	$3\dfrac{\sin\theta}{\cos\theta} = 2\cos\theta \Rightarrow 3\sin\theta = 2\cos^2\theta$	B1	1	**ag** convincingly found
(b)	$\sin^2\theta + \cos^2\theta = 1$	M1		oe seen
	$3\sin\theta = 2(1 - \sin^2\theta)$	A1		
	$2\sin^2\theta + 3\sin\theta - 2 = 0$	A1	3	**ag** convincingly found
(c)(i)	Attempt to solve for sin θ	M1		M0 for verification
	$(2\sin\theta - 1)(\sin\theta + 2) = 0$	A1		oe eg use of the formula.
	Since $-1 \le \sin\theta \le 1$, the only possible value for sin θ is $\frac{1}{2}$	A1		**ag** convincingly found and explained
(ii)	$\theta = 0.5235\ldots$	B1		In (ii) accept 3sf and in terms of π; ft on their 0.5235…
	$\theta = 2.6179\ldots$	B1√	2	
(d)	$3\tan 2x = 2\cos 2x \Rightarrow \sin 2x = \frac{1}{2}$	M1		Links with previous parts
	$2x = 30°$ or $150°$			
	$\Rightarrow x = 15°$	A1		
	or $x = 75°$	A1	3	
	Total		**12**	
9(a)(i)	$h = 0.5$	B1		
	Integral = $h/2$ {……}			
	{….}= $f(0) + 2[f\left(\frac{1}{2}\right) + f(1) + f\left(1\frac{1}{2}\right)] + f(2)$	M1		
	{….}=1+2[2+4+8]+16	A1		
	Integral =11.25	A1	4	
(ii)	Relevant trapezia drawn on a copy of given graph.	M1		
	Overestimate	A1`	2	
(b)	Stretch in y-direction	M1		
	scale factor 5	A1	2	
(c)	Sketch showing the reflection of the graph of the given curve in the y-axis	B1	1	
(d)(i)	$5(4^x) = 4^{-x} \Rightarrow 5(4^x) \times 4^x = 1$	M1		
	$\Rightarrow 5 \times 4^{2x} = 1 \Rightarrow 4^{2x} = 0.2$	A1	2	**ag** convincingly found
(ii)	$\ln 4^{2x} = \ln 0.2$	M1		oe using base 10
	$2x = \dfrac{\ln 0.2}{\ln 4}$	A1		
	$x = -\,0.58048(20\ldots)$	A1	3	Need 5sf or better
	Total		**14**	
	TOTAL		**75**	

General Certificate of Education
Specimen Unit
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

In addition to this paper you will require:
• an 8-page answer book;
• the AQA booklet of formulae and statistical tables;
• an insert for use in Question 5 (enclosed).
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Find $\dfrac{dy}{dx}$ when:

(a) $y = x\tan 3x$; *(3 marks)*

(b) $y = \dfrac{\sin x}{x}$. *(3 marks)*

2 A curve has equation $y = \dfrac{x}{\sqrt{(x^3+2)}}$. The region R is bounded by the curve, the x-axis from the origin to the point $(1,0)$ and the line $x = 1$.

(a) Explain why R lies entirely above the x-axis. *(1 mark)*

(b) Use Simpson's Rule with five ordinates (four strips) to find an approximation for the area of R, giving your answer to 3 significant figures. *(4 marks)*

(c) Find the exact value of the volume of the solid formed when R is rotated through 2π radians about the x-axis. *(5 marks)*

3 A curve has equation $y = e^{2x} - 4x$.

(a) Show that the x-coordinate of the stationary point on the curve is $\frac{1}{2}\ln 2$. Find the corresponding y-coordinate in the form $a + b\ln 2$, where a and b are integers to be determined. *(6 marks)*

(b) Find an expression for $\dfrac{d^2y}{dx^2}$ and hence determine the nature of the stationary point. *(3 marks)*

(c) Show that the area of the region enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 1$ is $\frac{1}{2}\left(e^2 - 5\right)$. *(5 marks)*

4 (a) Describe a sequence of geometrical transformations that maps the graph of $y = \sin x$ onto the graph of $y = 3 + \sin 2x$. *(4 marks)*

(b) Find the gradient of the curve with equation $y = 3 + \sin 2x$ at the point where $x = \frac{\pi}{6}$. *(3 marks)*

(c) (i) Find $\int x \sin 2x \, \mathrm{d}x$. *(4 marks)*

(ii) Hence show that $\int_0^{\frac{\pi}{2}} x(3 + \sin 2x) \, \mathrm{d}x = \frac{\pi(3\pi + 2)}{8}$. *(2 marks)*

5 [An insert is provided for use in answering this question.]

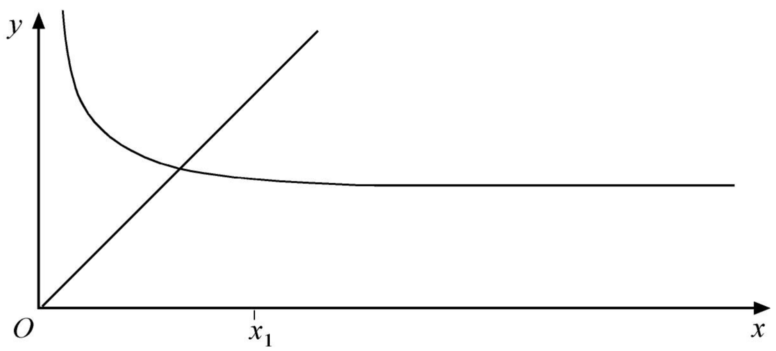
The curve with equation $y = x^3 - 4x^2 - 4$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 4 and 5. *(2 marks)*

(b) Show that the equation $x^3 - 4x^2 - 4 = 0$ can be rearranged in the form $x = 4 + \frac{4}{x^2}$. *(2 marks)*

(c) (i) Use the iterative formula $x_{n+1} = 4 + \frac{4}{x_n^2}$ with $x_1 = 5$ to find x_3, giving your answer to three significant figures. *(3 marks)*

(ii) The sketch shows the graphs of $y = 4 + \frac{4}{x^2}$ and $y = x$ and the position of x_1.
On the insert provided, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3. *(3 marks)*



Turn over ►

6 Solve the equation

$$4\cot^2 x + 12\operatorname{cosec} x + 1 = 0$$

giving all values of x to the nearest degree in the interval $0 \leqslant x \leqslant 360^\circ$. *(7 marks)*

7 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \frac{4}{3+x}, \quad x > 0$$

$$\mathrm{g}(x) = 9 - 2x^2, \quad x \in 3$$

(a) Find $\mathrm{fg}(x)$, giving your answer in its simplest form. *(2 marks)*

(b) (i) Solve the equation $\mathrm{g}(x) = 1$. *(2 marks)*

(ii) Explain why the function g does not have an inverse. *(1 mark)*

(c) Solve the equation $|\mathrm{g}(x)| = 1$. *(3 marks)*

(d) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$. *(4 marks)*

END OF QUESTIONS

Surname					Other Names					
Centre Number						Candidate Number				
Candidate Signature										

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General Certificate of Education
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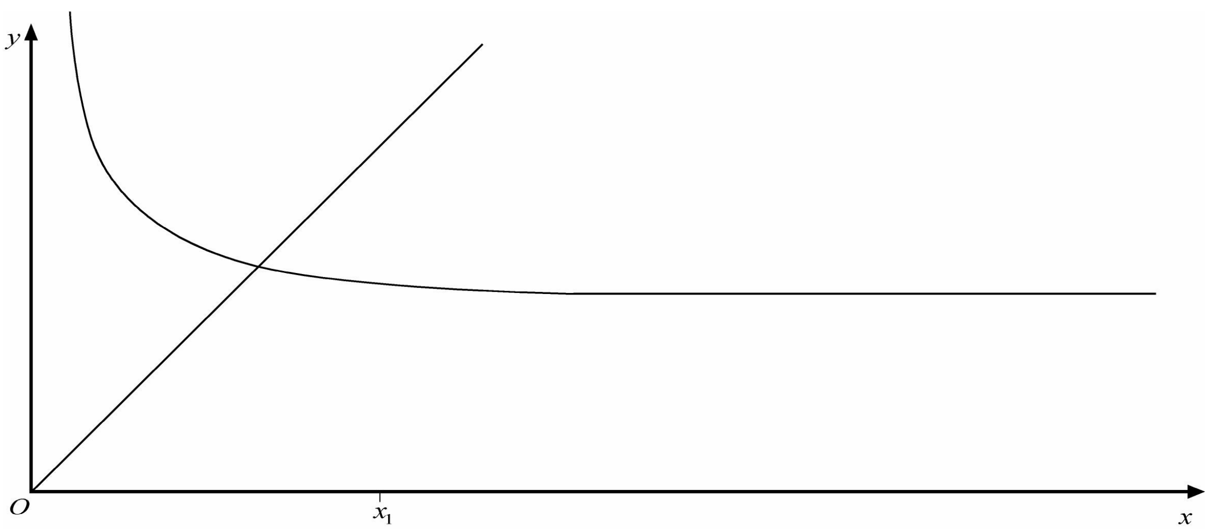
Advanced Level Examination

MATHEMATICS **MPC3**
Unit Pure Core 3

Insert for use in answering Question 5.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.



MPC3 Specimen

Question	Solution	Marks	Total	Comments
1(a)	$3x\sec^2 3x + \tan 3x$	M1		Product Rule
		M1		$\sec^2$
		A1	3	Correct
(b)	$\dfrac{x\cos x - \sin x}{x^2}$	M1		Quotient Rule
		B1		$\dfrac{\mathrm{d}(\sin x)}{\mathrm{d}x} = \cos x$
		A1	3	Correct
	Total		**6**	
2(a)	$y \geq 0$ when $x \geq 0$ so R is above x-axis	E1	1	
b)	"Outside multiplier" $\dfrac{1}{3} \times 0.25$	B1		
	$\dfrac{1}{3} \times 0.25\{y(0) + y(1) + 4[y(0.25) + y(0.75)] + 2y(0.5)\}$	M1		$y(0) = 0$; $y(0.25) = 0.17609$; $y(0.5) = 0.34300$; $y(0.75) = 0.48193$; $y(1) = 0.57735$
	= 0.3246193….	A1		Correct to at least 2 sf
	= 0.325 to 3 sf	A1	4	
(c)	$V = \pi\displaystyle\int_0^1 \frac{x^2}{x^3+2}\,\mathrm{d}x$	B1		
	$k\ln(x^3+2)$	M1		
	$\frac{1}{3}\ln(x^3+2)$	A1		Integration correct
	$\dfrac{\pi}{3}(\ln 3 - \ln 2)$	m1		Correct use of limits
		A1	5	
	Total		**10**	

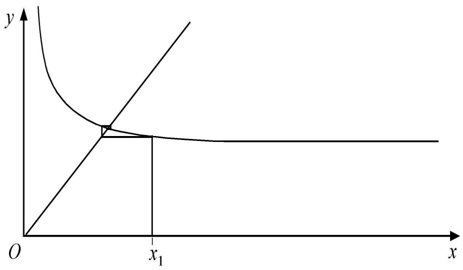
MPC3 (cont)

Question	Solution	Marks	Total	Comments
3(a)	$\frac{dy}{dx} = 2e^{2x} - 4$	M1 A1		ke^{2x}
	$\frac{dy}{dx} = 0 \Rightarrow e^{2x} = 2$	M1		
	$2x = \ln 2$			
	$\Rightarrow x = \frac{1}{2}\ln 2$	A1		**ag**
	$y = 2 - 2\ln 2 \quad a = 2$	B1		
	$b = -2$	B1	6	
(b)	$\frac{d^2y}{dx^2} = 4e^{2x}$	B1√		
	$x = \frac{1}{2}\ln 2 \qquad \frac{d^2y}{dx^2} = 8$	M1		
	$\Rightarrow$ Minimum Point	A1√	3	ft if negative for maximum
(c)	$\frac{1}{2}e^{2x} - 2x^2$	M1 A1 A1		ke^{2x} $\frac{1}{2}e^{2x}$ $-2x^2$
	$\left(\frac{1}{2}e^2 - 2\right) - \left(\frac{1}{2} - 0\right)$	m1		Use of limits 0 and 1
	$= \frac{1}{2}e^2 - 2\frac{1}{2} = \frac{1}{2}\left(e^2 - 5\right)$	A1	5	**ag**
	Total		**14**	

MPC3 (cont)

Question	Solution	Marks	Total	Comments
4(a)	One way stretch in $x-$direction	M1		
	scaling factor $\frac{1}{2}$	A1		
	translation (in y-direction)	M1		
	of $\begin{bmatrix}0\\3\end{bmatrix}$	A1	4	
(b)	$\frac{dy}{dx}=2\cos 2x$	M1 A1		cos Correct
	When $x=\frac{\pi}{6}$, $\frac{dy}{dx}=2x\frac{1}{2}=1$	A1	3	
(c)(i)	$\left[-\frac{x}{2}\cos 2x\right]$	M1 A1		Integration by parts attempt $-\frac{1}{2}\cos 2x$
	$+\int\frac{1}{2}\cos 2x\,dx$	A1		
	$=-\frac{x}{2}\cos 2x+\frac{1}{4}\sin 2x$	A1√	4	(ignore +c)
(ii)	$\frac{3x^2}{2}+$ previous result and attempt at limits	M1		Ignore $-[0]$
	$\frac{3\pi^2}{8}+\left(-\frac{-\pi}{4}\right)=\frac{3\pi^2}{8}+\frac{\pi}{4}$			
	$=\frac{\pi(3\pi+2)}{8}$	A1	2	**ag** all integration must be correct for A1
	Total		**13**	

MPC3 (cont)

Question	Solution	Marks	Total	Comments
5(a)	$f(4) = -4$; $f(5) = 21$	M1		$f(x) = x^3 - 4x^2 - 4$
	change of sign $\Rightarrow$ root between 4 and 5	A1	2	
(b)	$x - 4 - \frac{4}{x^2} = 0$	M1		Divide by x^2
	$\Rightarrow x = 4 + \frac{4}{x^2}$	A1	2	**ag**
(c)(i)	$x_2 = 4 + \frac{4}{5^2}$	M1		
	$= 4.16$	A1		
	$\Rightarrow x_3 = 4.23$ (to 3 sf)	A1	3	
(ii)		M1 A1 A1	3	Cobweb 'to curve first' Thin line $\Rightarrow x_2$ marked Next iteration $\Rightarrow x_3$ marked
	Total		**10**	
6	$4(\text{cosec}^2 x - 1) + 12\ \text{cosec}\, x + 1 = 0$	M1		Attempt at $\cot^2 x = \text{cosec}^2 x - 1$
	$4\ \text{cosec}^2 x + 12\ \text{cosec}\, x - 3 = 0$	A1		**or** may use $\cos^2 x = 1 - \sin^2 x$ after $4\frac{\cos^2 x}{\sin^2 x} + \frac{12}{\sin x} + 1 = 0$ etc
	$\text{cosec} x = \frac{-12 \pm \sqrt{192}}{8}$	M1		
	$= 0.23205 - 3.23205$	A1		
	$\text{cosec}\, x = \frac{1}{\sin x}$	B1		
	$\sin x = -0.3094$			
	$x = 198°$	A1		
	$342°$	A1	7	
	Total		**7**	

MPC3 (cont)

Question	Solution	Marks	Total	Comments
7(a)	$\mathrm{fg}(x)=\dfrac{4}{3+9-2x^2}$	M1		
	$=\dfrac{2}{6-x^2}$	A1	2	and no further wrong 'simplification'
(b)(i)	$9-2x^2=1 \Rightarrow x^2=4$	M1		$x=2$ scores M1 only
	$x=\pm 2$	A1	2	
(ii)	Two values of x map onto one $\Rightarrow$ not one-one	E1	1	**or** many-one
(c)	Two values from $\mathrm{g}(x)=1$ $\quad x=\pm 2$	B1√		
	$9-2x^2=-1 \Rightarrow x^2=5$	M1		
	$x=\pm\sqrt{5}$	A1	3	
(d)(i)	$y=\dfrac{4}{3+x} \Rightarrow 3y+yx=4$	M1		multiplying out and attempt to make x the subject
	$x=\dfrac{4}{y}-3$	A1		
	$\mathrm{f}^{-1}(x)=\dfrac{4}{x}-3$	A1	3	
(ii)	their $\mathrm{f}^{-1}(x)=\mathrm{f}(x)$ and multiplying up	M1		**or** $\mathrm{f}(x)=x$ $\Rightarrow 4=x(3+x)$
	$x^2+3x-4=0$	A1		$(x+4)(x-1)=0$
	$\Rightarrow x=-4,\ 1$	A1		
	$x>0$ $\Rightarrow$ only solution is $x=1$	A1	4	
	Total		**15**	
	TOTAL		**75**	

General Certificate of Education
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Advanced Level Examination

MATHEMATICS **MPC4**
Unit Pure Core 4

In addition to this paper you will require:
- an 8-page answer book;
- the AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The polynomials f(x) and g(x) are defined by

$$f(x) = 4x^2 + 4x - 3$$
$$g(x) = 4x^3 - x$$

(a) By considering $f\left(\frac{1}{2}\right)$ and $g\left(\frac{1}{2}\right)$, or otherwise, show that $f(x)$ and $g(x)$ have a common linear factor. *(3 marks)*

(b) Hence write $\dfrac{f(x)}{g(x)}$ as a simplified algebraic fraction. *(3 marks)*

2 (a) Obtain the binomial expansion of $(1+x)^{\frac{1}{2}}$ as far as the term in x^2. *(2 marks)*

(b) (i) Hence, or otherwise, find the series expansion of $(4+2x)^{\frac{1}{2}}$ as far as the term in x^2. *(3 marks)*

(ii) Find the range of values of x for which this expansion is valid. *(1 mark)*

3 (a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R is a positive constant and $0^\circ < \alpha < 90^\circ$.

Give the value of α to the nearest 0.1°. *(3 marks)*

(b) Hence find the solutions in the interval $0^\circ < \theta < 360^\circ$ of the equation

$$4\sin\theta - 3\cos\theta = 2$$

Give each solution to the nearest degree. *(4 marks)*

4 (a) Express $4\sin^2 x$ in the form $a + b\cos 2x$, where a and b are constants. *(2 marks)*

(b) Find the value of $\int_0^{\frac{\pi}{12}} 4\sin^2 x \, dx$. *(4 marks)*

(c) Hence find the volume generated when the part of the graph of

$$y = 2\sin x$$

between $x = 0$ and $x = \frac{\pi}{12}$ is rotated through one revolution about the x-axis. *(2 marks)*

5 The population P of a particular species is modelled by the formula

$$P = \mathrm{A}e^{-kt}$$

where t is the time in years measured from a date when $P = 5000$.

(a) Write down the value of A. *(1 mark)*

(b) Given that $P = 3500$ when $t = 10$, show that $k \approx 0.03567$. *(4 marks)*

(c) Find the value of the population 20 years after the initial date. *(3 marks)*

6 (a) Solve the differential equation

$$\frac{dx}{dt} = \frac{10 - x}{5}$$

given that $x = 1$ when $t = 0$. *(4 marks)*

(b) Find the value of t for which $x = 2$, giving your answer to three decimal places. *(2 marks)*

Turn over ►

7 (a) Express

$$\frac{25x+1}{(2x-1)(x+1)^2}$$

in the form

$$\frac{A}{2x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

(4 marks)

(b) Hence find the value of

$$\int_1^2 \frac{25x+1}{(2x-1)(x+1)^2}\,\mathrm{d}x$$

giving your answer in the form $p+q\ln 2$. *(6 marks)*

8 A curve is defined by the parametric equations

$$x=t^2+\frac{2}{t},\ y=t^2-\frac{2}{t},\ t\neq 0$$

(a) (i) Express $x+y$ and $x-y$ in terms of t. *(2 marks)*

(ii) Hence verify that the cartesian equation of the curve is

$$(x+y)(x-y)^2=32$$

(2 marks)

(b) (i) By finding $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$, calculate the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point for which $t=2$. *(5 marks)*

(ii) Hence find the equation of the tangent to the curve at this point.
Give your answer in the form $ax+by=c$, where a, b and c are integers. *(3 marks)*

9 The line l_1 has equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} + t\begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$.

The line l_2 has equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -4 \\ 0 \end{bmatrix} + s\begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$.

(a) Show that the lines l_1 and l_2 intersect and find the coordinates of their point of intersection. *(5 marks)*

(b) The point P on the line l_1 is where $t = p$, and the point Q has coordinates (5, 9, 11).

(i) Show that

$$\overrightarrow{QP} \cdot \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix} = 41p - 82$$

(4 marks)

(ii) Hence find the coordinates of the foot of the perpendicular from the point Q to the line l_1.

(3 marks)

END OF QUESTIONS

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MPC4 Specimen

Question	Solution	Marks	Total	Comments
1(a)	$f(\frac{1}{2}) = 0$, $g(\frac{1}{2}) = 0$	M1A1		or other complete method
	So $2x - 1$ is a common factor	A1	3	or $x - \frac{1}{2}$
(b)	$f(x) = (2x-1)(2x+3)$	B1		
	$g(x) = (2x-1)(2x^2+x)$	B1		
	So $\dfrac{f(x)}{g(x)} = \dfrac{2x+3}{2x^2+x}$	B1√	3	ft numerical error
	Total		**6**	
2(a)	$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots$	M1A1	2	M1 if two terms correct
(b)(i)	$(4+2x)^{\frac{1}{2}} = 2(1+\frac{1}{2}x)^{\frac{1}{2}}$	B1		
		M1		Reasonable attempt
	$\ldots = 2 + \frac{1}{2}x - \frac{1}{16}x^2 + \ldots$	A1	3	
(ii)	Valid if $-2 < x < 2$	B1	1	
	Total		**6**	
3(a)	$R = 5$	B1		
	$\cos\alpha$ or $\sin\alpha = \frac{4}{5}$ or $\frac{3}{5}$	M1		
	$\alpha \approx 36.9°$	A1	3	
(b)	$\sin(\theta - \alpha) = \frac{2}{5}$	M1		PI
	One solution is $\alpha + \sin^{-1}\frac{2}{5}$	m1		
	Solutions 60° and 193°	A1A1	4	Accept awrt 60 or 61, and awrt 193
	Total		**7**	
4(a)	Use of $\cos 2A \equiv 1 - 2\sin^2 A$	M1		PI
	$4\sin^2 x \equiv 2 - 2\cos 2x$	A1	2	
(b)	$\int \ldots\, dx = 2x - \sin 2x \ (+\, c)$	M1A1		M1 if at least one term correct
	Use of $\sin\frac{\pi}{6} = \frac{1}{2}$	m1		
	$\int_0^{\pi/12} \ldots\, dx = \dfrac{\pi}{6} - \dfrac{1}{2}$	A1	4	Accept 0.0236
(c)	$(2\sin x)^2 = 4\sin^2 x$	B1		
	So volume is $\pi\left(\frac{\pi}{6} - \frac{1}{2}\right)$	B1√	2	Accept 0.0741 ft one error
	Total		**8**	
5(a)	$A = 5000$	B1	1	
(b)	$3500 = 5000e^{-10k}$	B1√		oe; ft wrong value for A
	$\ln 0.7 = -10k$	M1		oe
	$k = -\frac{1}{10}\ln 0.7$	A1√		ft one numerical error
	$\ldots \approx 0.03567$	A1	4	convincingly shown (**ag**)
(c)	$P = 5000(0.7)^2$ or $5000e^{-0.7134}$	M1A1		M1 if only one small error
	$\ldots = 2450$	A1	3	Accept awrt 2450
	Total		**8**	

MPC4 (cont)

Question	Solution	Marks	Total	Comments
6(a)	Attempt to separate variables	M1		
	$\int \frac{dx}{10-x}\ dx = \pm k \ln(10-x)\ (+c)$	m1		
	$t = -5\ln(10-x) + c$	A1		
	$c = 5 \ln 9$	A1	4	
(b)	$x = 2 \Rightarrow\ t = 5 \ln 9 - 5 \ln 8$	M1		
	$\ldots \approx 0.589$	A1	2	
	Total		**6**	
7(a)	$25x+1 \equiv A(x+1)^2 + B(2x-1)(x+1) + C(2x-1)$	B1		
	$A = 6,\ B = -3,\ C = 8$	M1A2	4	A1 if only one error
(b)	$\int \ldots = 3\ln(2x-1)\ldots$	M1		
		A1√		ft wrong values for
	$\ldots - 3\ln(x+1) - \frac{8}{x+1}\ (+c)$	B1√		coeffs throughout this question
		B1√		
	$\int_1^2 \ldots = 3\ln 3 - 3\ln 3 + 3\ln 2 - (\frac{8}{3} - 4)$	m1		
	$\ldots = \frac{4}{3} + 3\ln 2$	A1√	6	
	Total		**10**	
8(a)(i)	$x + y = 2t^2,\ x - y = 4/t$	B2	2	
(ii)	$(x+y)(x-y)^2 = (2t^2)(16/t^2)$	M1		
	$\ldots = 32$	A1	2	Convincingly shown (**ag**)
(b)(i)	$\frac{dx}{dt} = 2t - \frac{2}{t^2},\ \frac{dy}{dt} = 2t + \frac{2}{t^2}$	M1A1		
	Use of chain rule	m1		
	When $t = 2$, $\frac{dy}{dx} = \frac{9}{7}$	A2,1√	5	ft numerical or sign error
(ii)	The point is (5, 3)	B1		
	The tangent is $y - 3 = \frac{9}{7}(x-5)$	M1		
	ie $9x - 7y = 24$	A1√	3	ft one numerical error
	Total		**12**	

MPC4 (cont)

Question	Solution	Marks	Total	Comments
9(a)	$3 + 4t = 9 - s$ $-2 + 4t = -4 + 3s$ $1 + 3t = 2s$ Solve two to obtain $s = 2$, $t = 1$ Check in 3rd equation Point of intersection is (7, 2, 4)	M1 A1 m1 A1 A1F	 5	oe
(b)(i)	$\overrightarrow{QP} = \begin{bmatrix} 4p-2 \\ 4p-11 \\ 3p-10 \end{bmatrix}$	M1A1		
	$\overrightarrow{QP}.\begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix} = 4(4p-2)+4(4-11)+3(3-10)$ $\ldots = 41p - 82$	m1 A1	 4	 Convincingly shown (**ag**)
(ii)	$\overrightarrow{QP} \perp l_1 \Rightarrow 41p - 82 = 0$ $\ldots \Rightarrow p = 2$ Foot of perpendicular is (11, 6, 7)	M1 A1 A1	 3	
	Total		**12**	
	TOTAL		**75**	