



ASSESSMENT and
QUALIFICATIONS
ALLIANCE

General Certificate of Education

Mathematics – Decision

SPECIMEN UNITS AND MARK SCHEMES

ADVANCED SUBSIDIARY MATHEMATICS (5361)
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

ADVANCED MATHEMATICS (6361)
ADVANCED PURE MATHEMATICS (6366)
ADVANCED FURTHER MATHEMATICS (6371)

MATHEMATICS
Unit Decision 1

MD01

In addition to this paper you will require:

- an 8-page answer book;
 - the AQA booklet of formulae and statistical tables;
 - an insert for use in Questions 3 and 5 (enclosed).
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD01.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 Use a Shell sort algorithm to rearrange the following numbers into ascending order, showing the new arrangement after each pass.

14, 27, 23, 36, 25, 18, 16, 66

(6 marks)

- 2 Four people A, B, C and D are to be matched to four tasks 1, 2, 3 and 4.

A bipartite graph showing the possible allocation of people to jobs is shown in **Figure 1**.

An initial matching is shown in **Figure 2**.

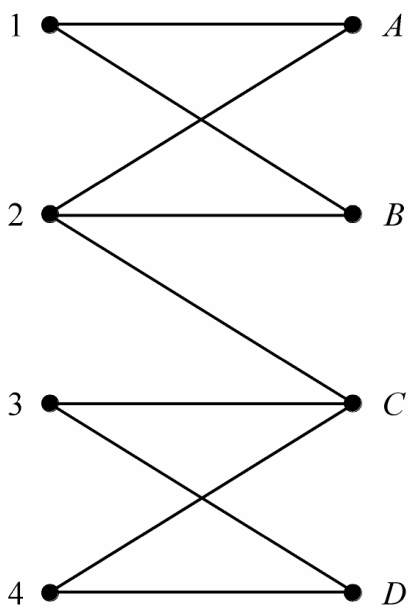


Figure 1

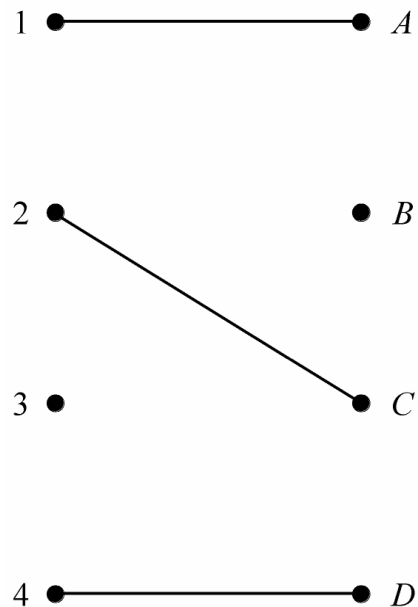


Figure 2

- (a) Write down an adjacency matrix that represents the bipartite graph shown in **Figure 1**. (2 marks)
- (b) There are four distinct alternating paths that can be generated from the initial matching shown in **Figure 2**.

One of the paths is

$$3 - C - 2 - A - 1 - B$$

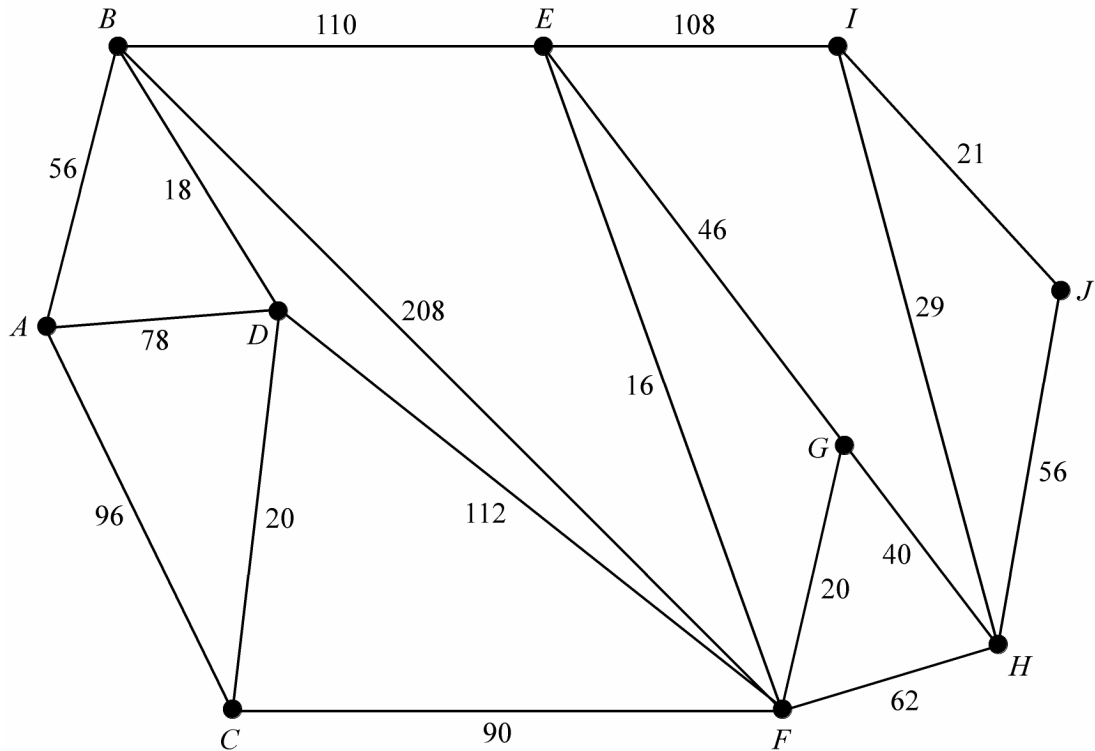
which produces the following complete matching

$$1 - B, 2 - A, 3 - C, 4 - D$$

- (i) Use the maximum matching algorithm from the initial matching to find another maximum matching, listing the complete matching generated. (3 marks)
- (ii) Find the remaining two alternating paths and list the complete matchings generated in each case. (4 marks)

3 [Figure 3, printed on the insert, is provided for use in answering this question.]

The following network shows the time, in minutes, to travel between ten towns.



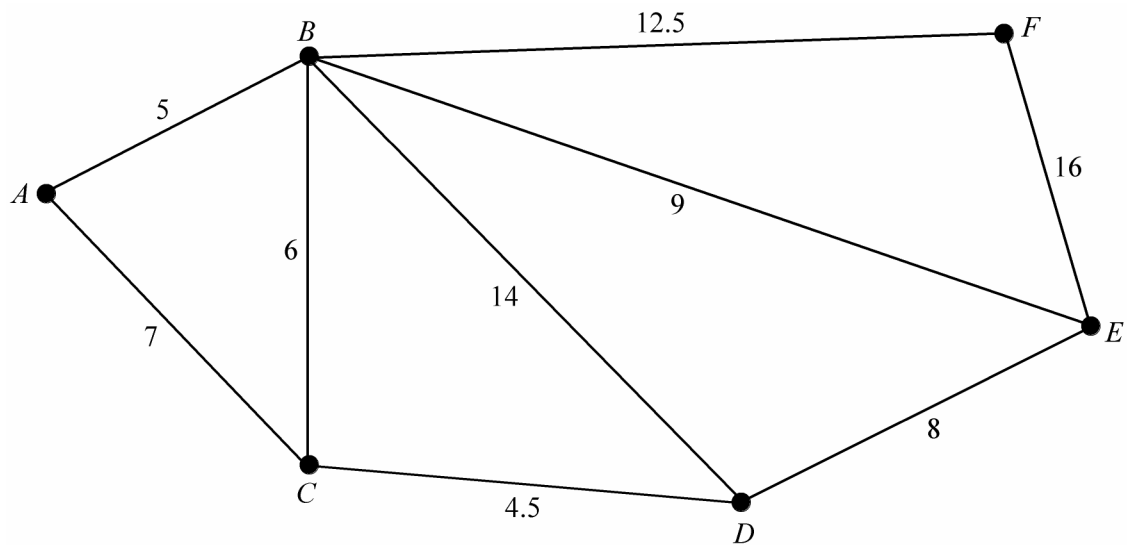
- (a) Use Dijkstra's algorithm on **Figure 3** to find the minimum time to travel from *A* to *J*, and state the route. (7 marks)
- (b) A new road is to be constructed connecting *D* to *E*. Find the time needed for travelling this section of road if the overall minimum journey time to travel from *A* to *J* is reduced by 10 minutes. State the new route. (3 marks)

TURN OVER FOR THE NEXT QUESTION

Turn over ►

4 A local council is responsible for gritting roads.

(a) The following diagram shows the lengths of roads, in miles, that have to be gritted.



The gritter is based at A and must travel along all the roads, at least once, before returning to A .

- (i) Explain why it is **not** possible to start from A and, by travelling along each road only once, return to A . *(1 mark)*
 - (ii) Find an optimal ‘Chinese postman’ route around the network, starting and finishing at A . State the length of your route. *(6 marks)*
- (b)
- (i) The connected graph of the roads in the area run by another council has six odd vertices. Find the number of ways of pairing these odd vertices. *(1 mark)*
 - (ii) For a connected graph with n odd vertices, find an expression for the number of ways of pairing these vertices. *(2 marks)*

5 [Figure 4, printed on the insert, is provided for use in answering this question.]

The Tony television company makes analogue and digital televisions. Both types of television require a number of component A and component B .

Each analogue television requires 2 of component A and 3 of component B .

Each digital television requires 4 of component A and 1 of component B .

Each day:

the company has 50 of component A and 24 of component B available;

and the company is to make at least 2 of each type of television, but no more than 20 in total.

The company sells each analogue television at a profit of £20 and each digital television at a profit of £25.

Each day the company makes and sells x analogue and y digital televisions.

The company needs to find its minimum and maximum total income, £ T .

- (a) Formulate the company's situation as a linear programming problem. *(5 marks)*
- (b) On **Figure 4**, draw a suitable diagram to enable the problem to be solved graphically, indicating the feasible region and the direction of the objective line. *(6 marks)*
- (c) Use your diagram to find the company's minimum and maximum daily income, £ T . *(6 marks)*

TURN OVER FOR THE NEXT QUESTION

Turn over ►

6 The following table shows the distances, in miles, between six stations.

	A	B	C	D	E	F
A	-	19	26	32	8	31
B	19	-	43	21	22	36
C	26	43	-	42	19	23
D	32	21	42	-	36	26
E	8	22	19	36	-	27
F	31	36	23	26	27	-

- (a) Use Prim's algorithm, starting from A, to find a minimum spanning tree for the network. *(3 marks)*
- (b) Roger is to visit each of the six stations. He decides to travel from one station to the next until he has visited all of the stations, starting and finishing at A.
- (i) Use the nearest neighbour algorithm, starting and finishing at A, to find an upper bound for the total distance Roger must travel. *(4 marks)*
- (ii) By initially ignoring A, find a lower bound for the total distance he must travel in visiting the six stations. *(5 marks)*
- (iii) Using your answer to parts (a) and (b), write down inequalities for M , the total distance in miles, that Roger has to travel. *(1 mark)*

7 A student is using the algorithm below to find the real roots of a quadratic equation.

```
LINE 10    INPUT A, B, C
LINE 20    D = B*B - 4*A*C
LINE 30    X1 = (-B + √D)/(2*A)
LINE 40    X2 = (-B - √D)/(2*A)
LINE 50    IF X1 = X2 THEN GOTO L
LINE 60    PRINT "DIFFERENT ROOTS", X1, X2
LINE 70    GOTO M
LINE 80    LABEL L
LINE 90    PRINT "EQUAL ROOTS", X1
LINE 100   LABEL M
LINE 110   END
```

(a) Trace the algorithm:

(i) if $A = 1$, $B = -4$, $C = 4$; *(2 marks)*

(ii) if $A = 2$, $B = 9$, $C = 9$; *(2 marks)*

(b) (i) Find a set of values of A , B and C for which the algorithm would fail. *(2 marks)*

(ii) Write down additional lines to ensure that the algorithm would not fail for **any** values of A , B and C that may be input. *(4 marks)*

END OF QUESTIONS

Surname						Other Names					
Centre Number						Candidate Number					
Candidate Signature											



General Certificate of Education
Specimen Unit
 Advanced Subsidiary Examination

MATHEMATICS
Unit Decision 1

MD01

Insert for use in answering Questions 3 and 5.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

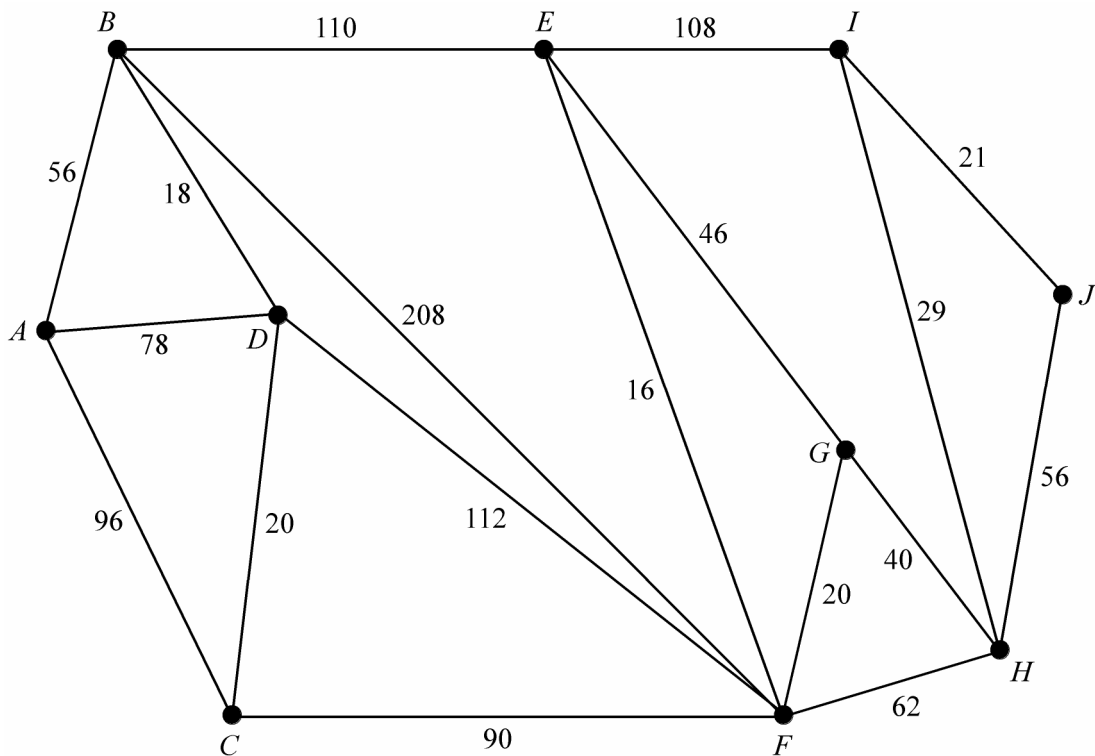


Figure 3 (for use in Question 3)

Turn over ►

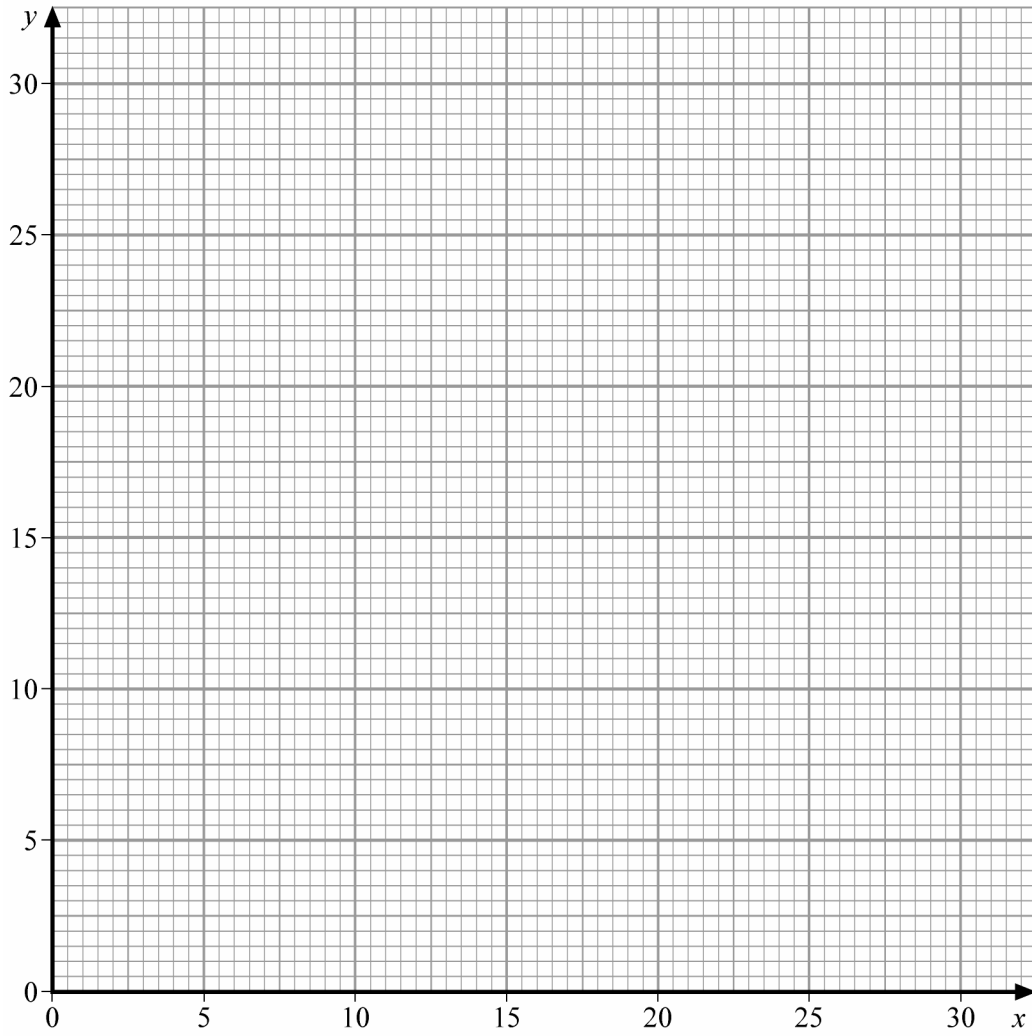


Figure 4 (for use in Question 5)

MD01 Specimen

Question	Solution	Marks	Total	Comments
1	$ \begin{array}{cccccccc} 14 & 27 & 23 & 36 & 18 & 25 & 16 & 66 \\ - & \sim & x & \bullet\bullet & - & \sim & x & \bullet\bullet \end{array} $	M1 A1		sca for comparing 27 & 25 / 16 & 23 All correct for 2 groups of 4 for 27 & 36 All correct
	$ \begin{array}{cccccccc} 14 & 25 & 16 & 36 & 18 & 27 & 23 & 66 \\ - & \sim & - & \sim & - & \sim & - & \sim \end{array} $	A1		
	$ \begin{array}{cccccccc} 14 & 25 & 16 & 27 & 18 & 36 & 23 & 66 \end{array} $	M1		
	$ \begin{array}{cccccccc} 14 & 16 & 18 & 23 & 25 & 27 & 36 & 66 \end{array} $	A1 A1		
	Total		6	
2 (a)	$ \begin{array}{cccc} & A & B & C & D \\ 1 & \left(\begin{array}{cccc} 1 & 1 & 0 & 0 \end{array} \right) \\ 2 & \left(\begin{array}{cccc} 1 & 1 & 1 & 0 \end{array} \right) \\ 3 & \left(\begin{array}{cccc} 0 & 0 & 1 & 1 \end{array} \right) \\ 4 & \left(\begin{array}{cccc} 0 & 0 & 1 & 1 \end{array} \right) \end{array} $	M1A1	2	or answers from (ii) or in diagram or in diagram
(b)(i)	$3 - C - 2 - B$ Match (1A 2B 3C 4D)	M1A1 B1	3	
(ii)	$3 - D - 4 - C - 2 - B$ Match (1A 2B 3D 4C)	M1 B1	4	
	$3 - D - 4 - C - 2 - A - 1 - B$ Match (1B 2A 3D 4C)	M1 B1	4	
	Total		9	

MD01 (cont)

Question	Solution	Marks	Total	Comments
3(a)	<p>Route: <i>A B E F G H I J</i></p>	M1 A1 A1✓ A1F✓ A1 M1 A1✓	7	sca (oe) for <i>D</i> for <i>F</i> for <i>G</i> for <i>J</i> as 292 cao Going backwards
(b)	<p>To <i>E</i>, $74 + x$ $74 + x = 156$ $x = 82$</p> <p>Route: <i>A B D E F G H I J</i></p>	M1 A1✓ M1 A1✓ B1	3	for part (a)
Total			10	
4(a)(i)	Odd vertices \Rightarrow repeats	E1	1	
(ii)	<p>Odd <i>BCDE</i> $BC + DE = 14$ $BD + CE = 10.5 + 12.5 = 23$ $BE + CD = 9 + 4.5 = 13.5$ Dist $A = 13.5 + 82$ $= 95.5$ Route: <i>A B C D E F B E B D C A</i></p>	M1 m1 A1 M1 A1✓ B1	6	Considering odds for pairs oe
(b)(i)	$5 \times 3 \times 1 = 15$	B1	1	
(ii)	$(n-1)(n-3)(n-5)\dots \times 1$	M1 A1	2	
Total			10	

MD01 (cont)

Question	Solution	Marks	Total	Comments
5 (a)	$2x + 4y \leq 50$ $3x + y \leq 24$ $x + y \leq 20$ $x \geq 2, y \geq 2$ $(T =) 20x + 25y$	B1 B1 B1 B1 B1	5	oe (Strict inequalities -1) (equalities -1) (A & B -1) Both
(b)		B1 × 3 B1 B1 ✓ B1	6	3 lines $x = 2, y = 2$ (both) closed region marked Objective line
(c)	$T = 20x + 25y$ (Min at (2,2) =) £90 Max at (4.6, 10.2) Impossible Max = £335	M1 A1 B2,1,0 B2	6	Considering extreme points on their region, Considering (3,11) (4,10) (5,9) (B1 for 330, 325)
Total			17	

MD01 (cont)

Question	Solution	Marks	Total	Comments																								
6 (a)	AE 8 AB 19 EC 19 BD 21 CF 23 Total 90	M1 A1 B1	3	5 edges All correct																								
6 (b)(i)	A → E → C → F → D → B → A 8 19 23 26 21 19 Tour = 116	M1 M1 A1 B1	4	Tour starts and finishes at A Visits all vertices For correct order (EC 97 scores $\frac{2}{4}$)																								
(ii)	MST is B → D → E → C → F MST = 85 Minimum = (85) + 8 + 19 their (85) = 112	M1 A1 B1 M1 A1✓	5	For spanning tree For four edges																								
(iii)	(112) ≤ M ≤ (116)	B1✓	1	their (b)(i) and (b)(ii)																								
Total			13																									
7 (a)(i)	<table style="margin-left: 40px;"> <tr> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>X₁</td> <td>X₂</td> </tr> <tr> <td>1</td> <td>-4</td> <td>4</td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td>0</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>2</td> <td>2</td> </tr> </table>	A	B	C	D	X ₁	X ₂	1	-4	4							0							2	2	M1 A1	2	
A	B	C	D	X ₁	X ₂																							
1	-4	4																										
			0																									
				2	2																							
(ii)	<table style="margin-left: 40px;"> <tr> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>X₁</td> <td>X₂</td> </tr> <tr> <td>2</td> <td>9</td> <td>9</td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td>9</td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td>$-\frac{3}{2}$</td> <td>$-\frac{6}{2}$</td> </tr> </table>	A	B	C	D	X ₁	X ₂	2	9	9							9							$-\frac{3}{2}$	$-\frac{6}{2}$	M1 A1	2	oe
A	B	C	D	X ₁	X ₂																							
2	9	9																										
			9																									
				$-\frac{3}{2}$	$-\frac{6}{2}$																							
(b)(i)	Any values where D < 0 or A = 0	M1 A1	2	For attempt For correct values																								
(ii)	Line 25 IF D < 0 then print "NO SOLS" GOTO M Line 15 If A = 0 then print "NOT QUADRATIC" GOTO M	M1 A1 M1 A1	4	Attempt Attempt																								
Total			10																									
TOTAL			75																									

MATHEMATICS
Unit Decision 2

MD02

In addition to this paper you will require:

- an 8-page answer book;
- the AQA booklet of formulae and statistical tables;
- an insert for use in Questions 3 and 5 (enclosed);
- one sheet of graph paper for use in Question 4.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information

- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice

- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

- 1 The coach of a relay team has five athletes from which she is to choose four to run the four legs of a relay race. The time, in seconds, which the coach assumes each athlete will take to run each stage of the relay is shown in the following table.

Athlete	Relay stage			
	1	2	3	4
<i>A</i>	66	67	63	60
<i>B</i>	67	71	73	61
<i>C</i>	61	70	65	63
<i>D</i>	69	72	74	62
<i>E</i>	70	68	76	65

Use the Hungarian algorithm, reducing rows first then columns, to decide how the coach should choose the four athletes, one for each stage, to minimise the total time for the team. State the minimum time. *(9 marks)*

- 2 Over a three week period, a small plastics company is to prepare mouldings of three types of Christmas figure: Father Christmas (*F*), Reindeer (*R*) and Snowman (*S*). One moulding is to be prepared each week. The cost of preparing the three mouldings varies according to the mouldings previously prepared. The company wishes to calculate its maximum preparation costs.

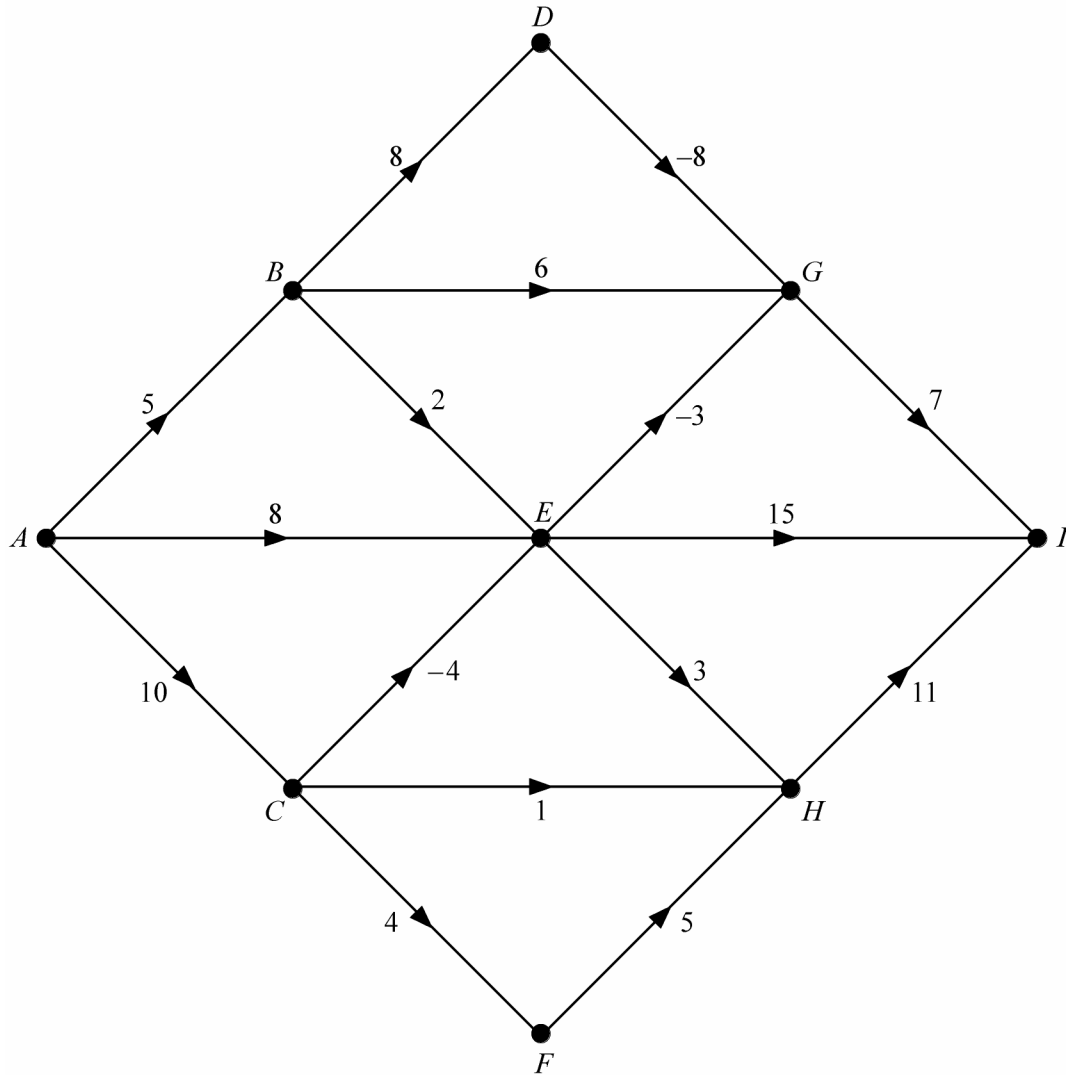
The costs, in pounds, are given in the table below.

Week	Previous moulding(s)	Cost (£000's)		
		<i>F</i>	<i>R</i>	<i>S</i>
1	-	330	360	390
2	<i>F</i>	-	300	330
	<i>R</i>	380	-	270
	<i>S</i>	400	290	-
3	<i>F</i> and <i>R</i>	-	-	300
	<i>F</i> and <i>S</i>	-	250	-
	<i>R</i> and <i>S</i>	270	-	-

Using dynamic programming, together with a labelled network or otherwise, determine the order of preparing the mouldings that maximises the total cost to the company. *(9 marks)*

3 [Figure 1, printed on a separate sheet, is provided for use in answering this question.]

The following network shows nine vertices. The number on each arc is the cost of a journey between the corresponding vertices.



Use dynamic programming on **Figure 1** to find the minimum cost of a route from A to I . State the route corresponding to this minimum cost. (7 marks)

TURN OVER FOR THE NEXT QUESTION

Turn over ►

4 [Graph paper is provided for use in answering this question.]

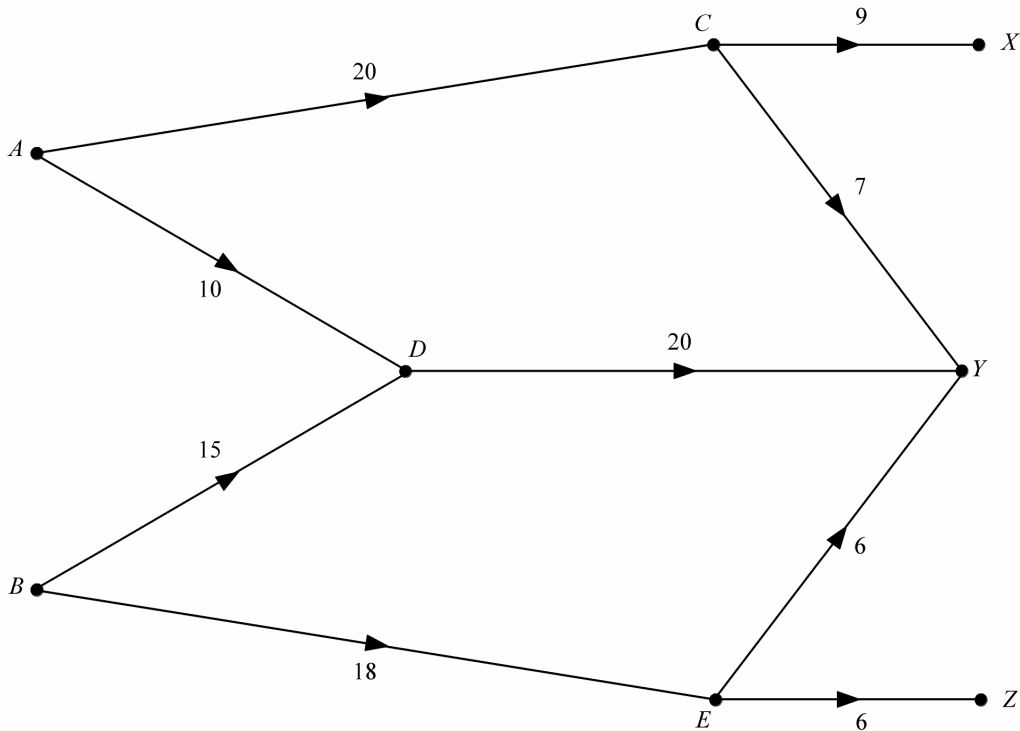
A small building project is to be undertaken. The following precedence table shows each activity, its duration, and the number of workers required to complete the activity.

Activity	Immediate predecessor	Duration (days)	Number of workers
<i>A</i>		3	4
<i>B</i>	<i>A</i>	4	2
<i>C</i>	<i>A</i>	2	1
<i>D</i>	<i>B</i>	3	2
<i>E</i>	<i>D</i>	11	4
<i>F</i>	<i>D</i>	4	2
<i>G</i>	<i>C,D</i>	5	2
<i>H</i>	<i>F,G</i>	2	1
<i>I</i>	<i>E,H</i>	2	4

- Construct an activity network for the project. (3 marks)
- Find the earliest start time for each activity. (2 marks)
- Find the latest finish time for each activity. (2 marks)
- State the float time for each non-critical activity. (2 marks)
- Given that each activity starts as early as possible, draw a resource histogram for the project. (4 marks)
- Given that there are only 4 workers available at any time, find the minimum overrun time for the project. (3 marks)

5 [Figures 2 and 3, printed on a separate sheet, are provided for use in answering this question.]

A greengrocer has two suppliers, A and B , and three storage depots, C , D and E . He needs to transport his stock to three retail outlets X , Y and Z . The capacities of the possible routes, in van loads per week, are shown in the following diagram.



- (a) Add a super-source (S) and a super-sink (W) on **Figure 2** to obtain a single source, single sink capacitated network. Show the capacities of each arc you have added. (2 marks)
- (b) State the maximum flow along the routes $SADYW$ and $SBEZW$. (2 marks)
- (c) (i) Show your answers to part (b) on **Figure 3** and, taking this as the initial flow pattern, use flow augmentation to find the maximum flow from S to W . (6 marks)
- (ii) Prove that your flow is maximal. (2 marks)

TURN OVER FOR THE NEXT QUESTION

Turn over ►

- 6 A linear programming problem in x and y is to be solved. Part of the initial tableau is given below.

x	y	r	s	t	
4	3	1	0	0	33
-1	1	0	1	0	4
2	5	0	0	1	27

- (a) In addition to $x \geq 0$ and $y \geq 0$, write down the **three** inequalities in this problem. (2 marks)
- (b) (i) The objective function $P = 2x + 2y$ is to be maximised. Solve this linear programming problem using the simplex algorithm, by initially using a value in the x column as the pivot. (You do **not** require more than two iterations.) (7 marks)
- (ii) State your final values of P , x and y . (2 marks)
- 7 Two people, A and B , play a zero sum game. The game is represented by the following pay-off matrix for A .

		B		
		I	II	III
A	I	5	1	3
	II	2	5	4
	III	4	-1	2

- (a) Show that there is no stable solution. (2 marks)
- (b) Explain why it will never be optimal for A to adopt strategy **III**. (1 mark)
- (c) By considering mixed strategies, and giving your answers as exact fractions:
- (i) find the optimal mixed strategy for A ; (7 marks)
- (ii) find the value of the game. (1 mark)

END OF QUESTIONS

Surname						Other Names				
Centre Number						Candidate Number				
Candidate Signature										

General Certificate of Education
Specimen Unit
Advanced Level Examination



MATHEMATICS
Unit Decision 2

MD02

Insert for use in answering Questions 3 and 5.

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Fasten this insert securely to your answer book.

FIGURE 1 FOR USE IN ANSWERING QUESTION 3

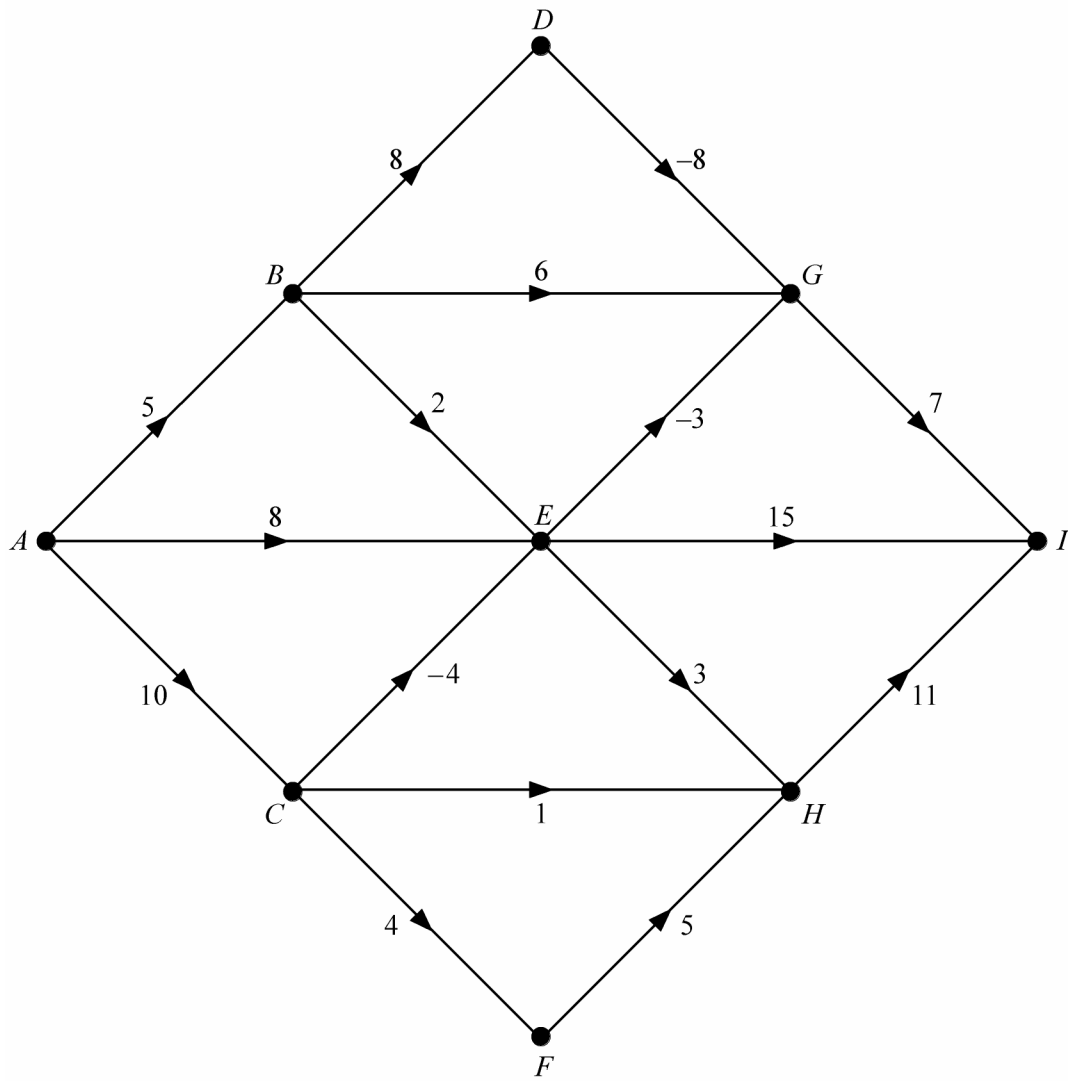


Figure 1

Turn over ►

FIGURE 2 FOR USE IN ANSWERING QUESTION 5(a)

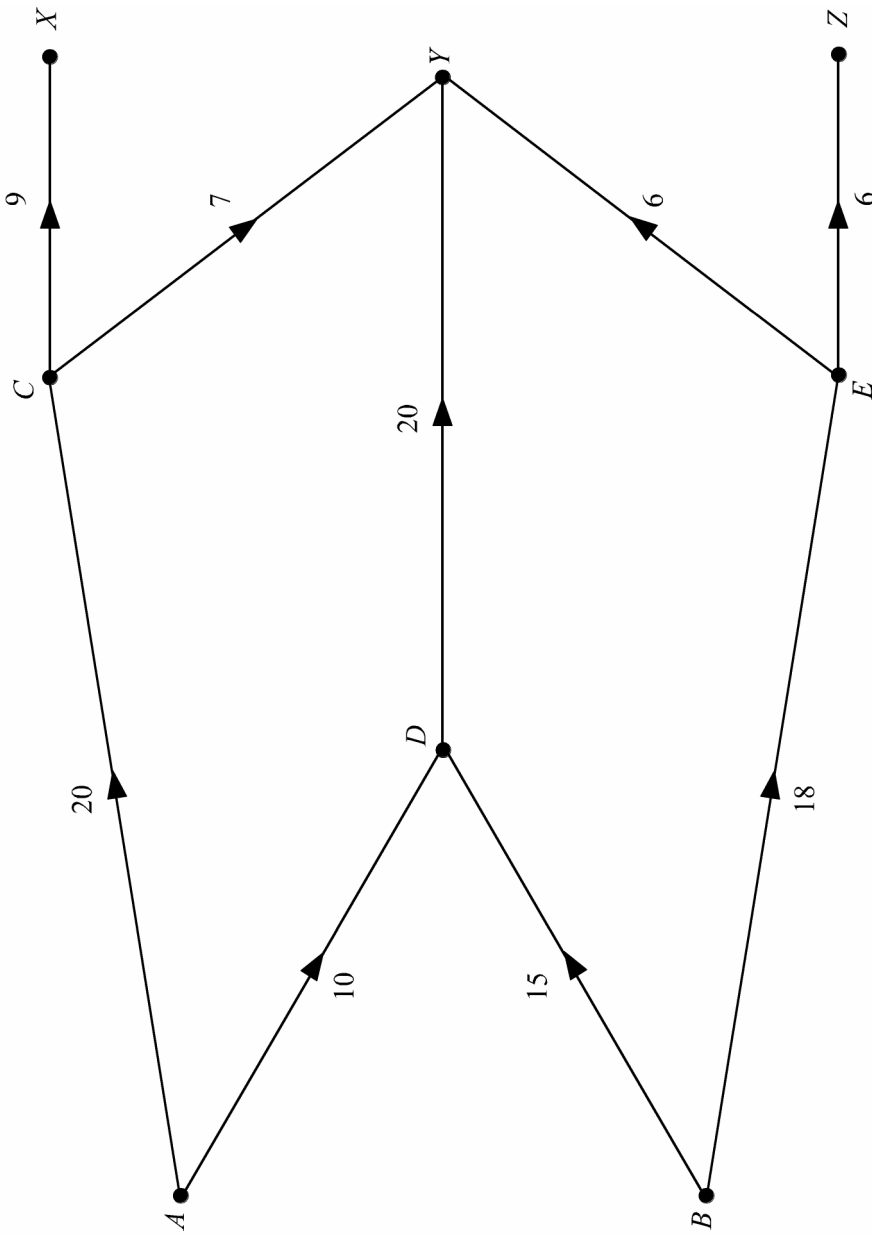
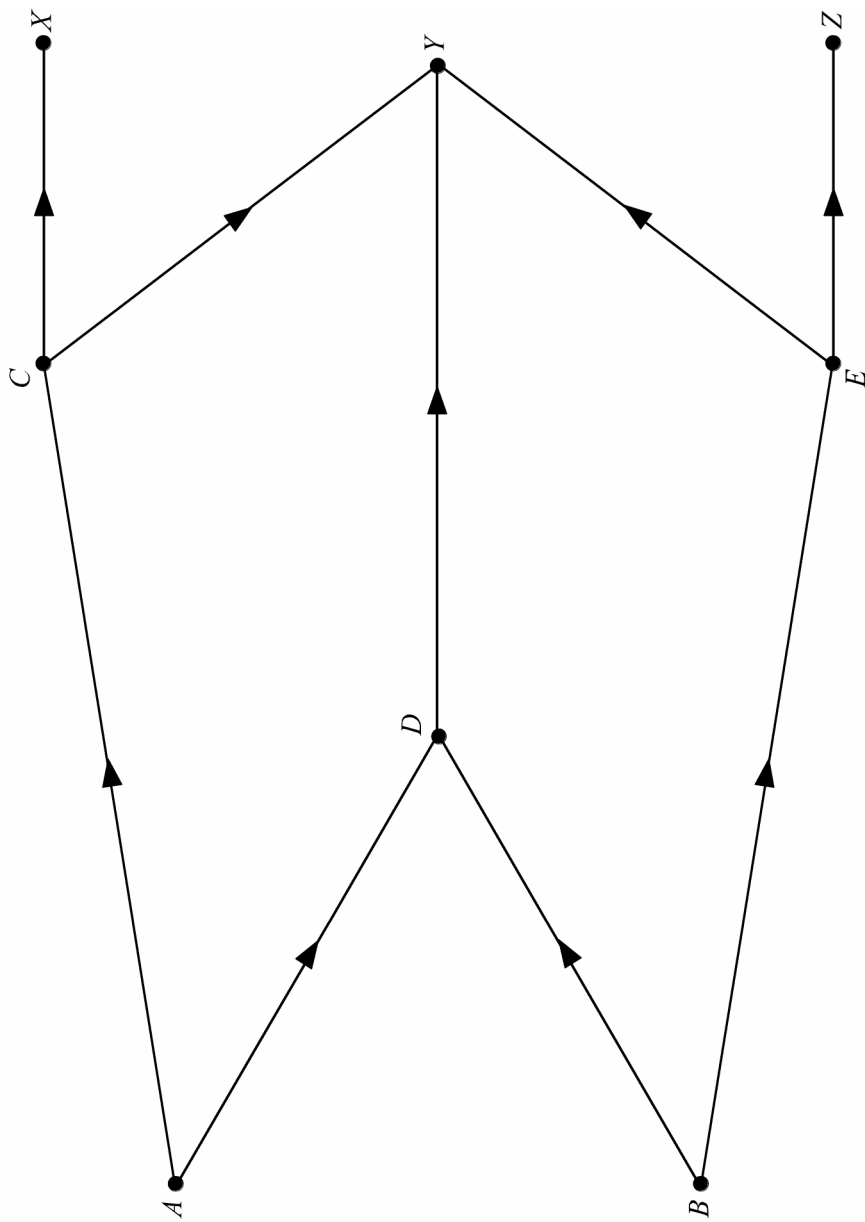


Figure 2

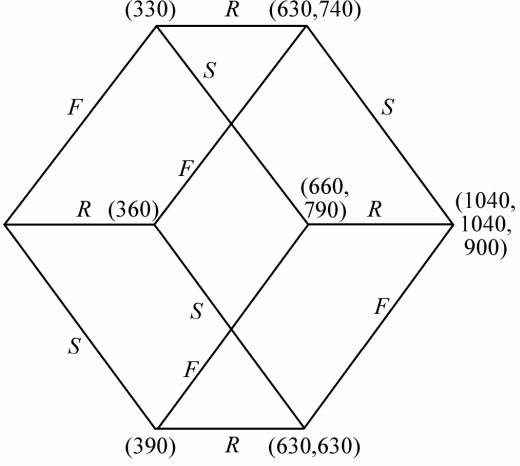
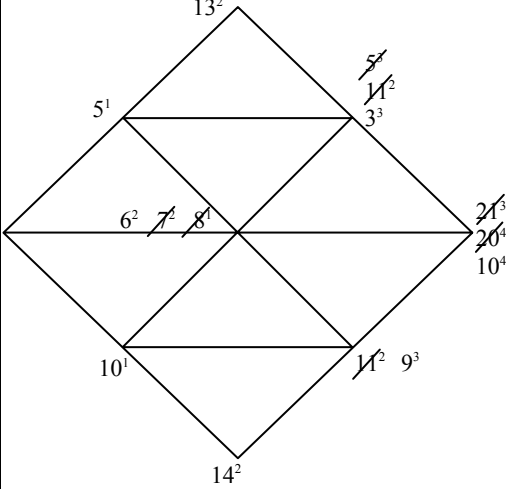
Turn over ►

FIGURE 3 FOR USE IN ANSWERING QUESTION 5(c)**Figure 3**

MD02 Specimen

Question	Solution	Marks	Total	Comments
1	<p>Add extra column of 76</p> <p>Reduce rows</p> $\begin{array}{r rrrrr} 60 & 6 & 7 & 3 & 0 & 16 \\ \hline 61 & 6 & 10 & 12 & 0 & 15 \\ \hline 61 & 0 & 9 & 4 & 2 & 15 \\ \hline 62 & 7 & 10 & 12 & 0 & 14 \\ \hline 65 & 5 & 3 & 11 & 0 & 11 \end{array}$ <p>Reduce columns</p> $\begin{array}{r rrrrr} 0 & 3 & 3 & 0 & 11 \\ \hline \cancel{6} & \cancel{4} & \cancel{0} & \cancel{0} & \cancel{5} \\ 6 & 7 & 9 & 0 & 4 \\ \hline \cancel{0} & \cancel{6} & \cancel{1} & \cancel{2} & \cancel{4} \\ 7 & 7 & 9 & 0 & 3 \\ \hline \cancel{5} & \cancel{0} & \cancel{8} & \cancel{0} & \cancel{0} \end{array}$ <p>4 rows, so adjust</p> $\begin{array}{r} 6 & 4 & \textcircled{0} & 3 & 5 \\ 3 & 4 & 6 & \textcircled{0} & 1 \\ \textcircled{0} & 6 & 1 & 5 & 4 \\ 4 & 4 & 6 & 0 & \textcircled{0} \\ 5 & \textcircled{0} & 8 & 3 & 0 \end{array}$ <p>Match <i>A - 3, B - 4, C - 1, D - 5, E - 2</i> Min = 253</p>	<p>M1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>9</p>	<p>This M1 A1 not available for candidates who reduce columns first.</p> <p>Max possible mark of 7 for candidates who reduce columns first</p>
Total			9	

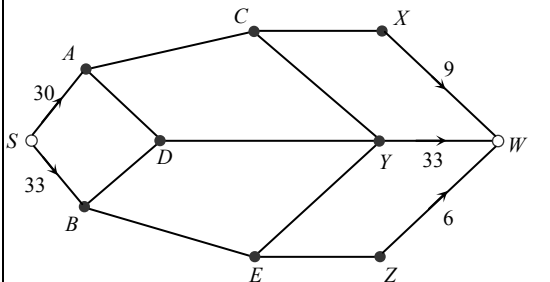
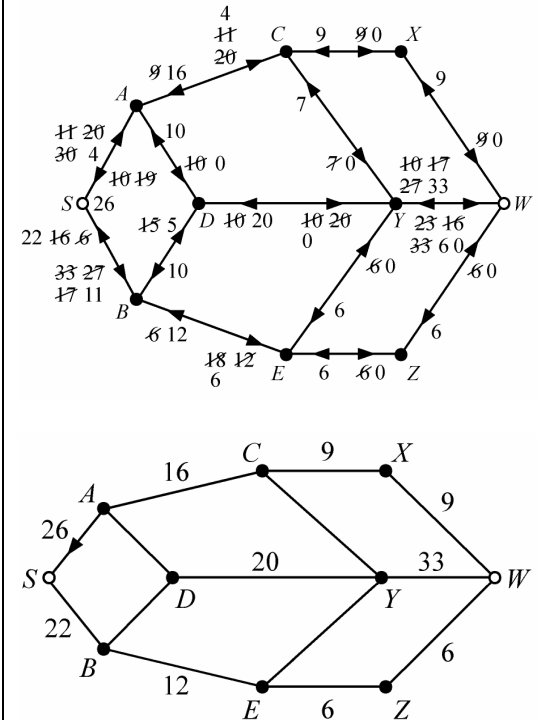
MD02 (cont)

Question	Solution	Marks	Total	Comments
2	 <p>Max = 1040 Route RFS or SFR</p>	<p>G1 M1</p> <p>M1 M1 A2, 1, 0</p> <p>B1 B1 +B1</p>	<p>9</p>	<p>Network diag sca</p> <p>3 pairs after W1 3 after W2 at stage 2</p> <p>Both</p>
Total			9	
3	 <p>Route <i>ACEGI</i> <i>I</i> = 10</p>	<p>M1</p> <p>A1</p> <p>A1✓</p> <p>A1✓</p> <p>A1✓</p> <p>B1</p> <p>B1</p>	<p>7</p>	<p>sca</p> <p>For <i>E</i></p> <p>For <i>G</i></p> <p>For <i>H</i></p> <p>For <i>I</i></p>
Total			7	

MD02 (cont)

Question	Solution	Marks	Total	Comments																														
4(a)		M1		sca																														
	<pre> graph LR A[A: 0 3] --> B[B: 3 7] A --> C[C: 3 14] B --> D[D: 7 10] B --> E[E: 10 21] C --> G[G: 10 19] D --> E D --> F[F: 10 19] E --> H[H: 15 21] E --> I[I: 21 23] F --> H G --> H H --> I </pre>	A2	3	-1 e.e.																														
(b)		M1		For forward pass																														
		A1	2	For all correct																														
(c)		M1		For back pass																														
		A1	2	For all correct																														
(d)	<p>C - 9 F - 5 G - 4 H - 4</p>	B1✓		For all four letters																														
		B1	2	For four correct floats																														
(e)	<table border="1"> <caption>Histogram Data</caption> <thead> <tr> <th>Activity</th> <th>Duration</th> <th>Frequency</th> </tr> </thead> <tbody> <tr><td>A</td><td>3</td><td>4</td></tr> <tr><td>B</td><td>4</td><td>3</td></tr> <tr><td>C</td><td>5</td><td>2</td></tr> <tr><td>D</td><td>6</td><td>2</td></tr> <tr><td>E</td><td>10</td><td>8</td></tr> <tr><td>F</td><td>11</td><td>6</td></tr> <tr><td>G</td><td>12</td><td>5</td></tr> <tr><td>H</td><td>13</td><td>4</td></tr> <tr><td>I</td><td>21</td><td>4</td></tr> </tbody> </table>	Activity	Duration	Frequency	A	3	4	B	4	3	C	5	2	D	6	2	E	10	8	F	11	6	G	12	5	H	13	4	I	21	4	M1		sca
Activity	Duration	Frequency																																
A	3	4																																
B	4	3																																
C	5	2																																
D	6	2																																
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F	11	6																																
G	12	5																																
H	13	4																																
I	21	4																																
		A3✓	4	-1 e.e.																														
(f)	<p>Only 4 works so problem at E F G Can't start F/G until E completed Overrun of 9, completion at 32</p>	M1																																
		A1																																
		A1	3																															
Total			16																															

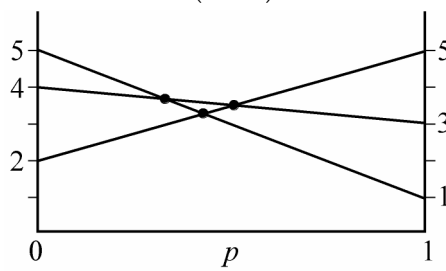
MD02 (cont)

Question	Solution	Marks	Total	Comments
5(a)		M1		For S and W
(b)	$SADYW = 10$ $SBEZW = 6$	B1 B1	2	For five arcs correct (\geq)
(c)(i)		M1 M1 A1 A1 A1	6	For starting with $SADYW, SBEZW$ sca For AC For SA For SB oe
(ii)	Maximum flow = 48 Minimum cut CX, CY, DY, EY, EZ $=48$	M1 A1	2	
	Total		12	

MD02 (cont)

Question	Solution	Marks	Total	Comments																																																																																																																																
6(a)	$4x + 3y \leq 33$ $-x + y \leq 4$ $2x + 5y \leq 27$	M1 A1	2																																																																																																																																	
(b)	$4^* \quad 3 \quad 1 \quad 0 \quad 0 \quad 0 \quad 33 \quad (R_1)$ $-1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 4 \quad (R_2)$ $2 \quad 5 \quad 0 \quad 0 \quad 1 \quad 0 \quad 27 \quad (R_3)$ $-2 \quad -2 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0$ Pivot about $x, 4$ $4 \quad 3 \quad 1 \quad 0 \quad 0 \quad 0 \quad 33 \quad R_1$ $0 \quad 7 \quad 1 \quad 4 \quad 0 \quad 0 \quad 49 \quad 4R_2+R_1$ $0 \quad 7^* \quad -1 \quad 0 \quad 2 \quad 0 \quad 21 \quad 2R_3-R_1$ $0 \quad -1 \quad 1 \quad 0 \quad 0 \quad 2 \quad 33 \quad 2R_4+R_1$ Pivot about $x, 7$ $28 \quad 0 \quad 10 \quad 0 \quad -6 \quad 0 \quad 168 \quad 7R_1-3R_3$ $0 \quad 0 \quad 2 \quad 4 \quad -2 \quad 0 \quad 28 \quad R_2-R_3$ $0 \quad 7 \quad -1 \quad 0 \quad 2 \quad 0 \quad 21 \quad R_3$ $0 \quad 0 \quad 6 \quad 0 \quad 2 \quad 14 \quad 252 \quad 7R_4+R_3$ All positive, therefore optimal	M1 m1 A1 M1 m1 A1 B1	7	<p>Alternative</p> <table border="1"> <thead> <tr> <th>P</th> <th>n</th> <th>y</th> <th>r</th> <th>s</th> <th>t</th> <th>u</th> <th></th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-2</td> <td>-2</td> <td>0</td> <td>0</td> <td>0</td> <td>0</td> <td></td> </tr> <tr> <td>0</td> <td>4</td> <td>3</td> <td>1</td> <td>0</td> <td>0</td> <td>33</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>M1</td> </tr> <tr> <td>0</td> <td>-1</td> <td>1</td> <td>0</td> <td>1</td> <td>0</td> <td>4</td> <td></td> </tr> <tr> <td>0</td> <td>2</td> <td>5</td> <td>0</td> <td>0</td> <td>1</td> <td>27</td> <td></td> </tr> <tr> <td>1</td> <td>0</td> <td>$-\frac{1}{2}$</td> <td>$\frac{1}{2}$</td> <td>0</td> <td>0</td> <td>$\frac{33}{2}$</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>M1</td> </tr> <tr> <td>0</td> <td>1</td> <td>$\frac{3}{4}$</td> <td>$\frac{1}{4}$</td> <td>0</td> <td>0</td> <td>$\frac{33}{4}$</td> <td>A1</td> </tr> <tr> <td>0</td> <td>0</td> <td>$\frac{7}{4}$</td> <td>$\frac{1}{4}$</td> <td>1</td> <td>0</td> <td>$\frac{49}{4}$</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>$\frac{7}{2}$</td> <td>$-\frac{1}{2}$</td> <td>0</td> <td>1</td> <td>$\frac{21}{2}$</td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>M1</td> </tr> <tr> <td>1</td> <td>0</td> <td>0</td> <td>$\frac{3}{7}$</td> <td>0</td> <td>$\frac{1}{7}$</td> <td>18</td> <td>m1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>$\frac{5}{14}$</td> <td>0</td> <td>$-\frac{3}{14}$</td> <td>6</td> <td></td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>$\frac{1}{2}$</td> <td>1</td> <td>$-\frac{1}{2}$</td> <td>7</td> <td>A1</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>$-\frac{1}{7}$</td> <td>0</td> <td>$\frac{2}{7}$</td> <td>3</td> <td>B1</td> </tr> </tbody> </table>	P	n	y	r	s	t	u		1	-2	-2	0	0	0	0		0	4	3	1	0	0	33									M1	0	-1	1	0	1	0	4		0	2	5	0	0	1	27		1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{33}{2}$									M1	0	1	$\frac{3}{4}$	$\frac{1}{4}$	0	0	$\frac{33}{4}$	A1	0	0	$\frac{7}{4}$	$\frac{1}{4}$	1	0	$\frac{49}{4}$		0	0	$\frac{7}{2}$	$-\frac{1}{2}$	0	1	$\frac{21}{2}$									M1	1	0	0	$\frac{3}{7}$	0	$\frac{1}{7}$	18	m1	0	1	0	$\frac{5}{14}$	0	$-\frac{3}{14}$	6		0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}$	7	A1	0	0	1	$-\frac{1}{7}$	0	$\frac{2}{7}$	3	B1
P	n	y	r	s	t	u																																																																																																																														
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0	0	0	$\frac{1}{2}$	1	$-\frac{1}{2}$	7	A1																																																																																																																													
0	0	1	$-\frac{1}{7}$	0	$\frac{2}{7}$	3	B1																																																																																																																													
(c)	$14P = 252$ $P = 18$ $y = 3, \quad x = 6$	B1 B1	2	For both																																																																																																																																
Total			11																																																																																																																																	

MD02 (cont)

Question	Solution	Marks	Total	Comments																
7(a)	<table style="display: inline-table; border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px;">5</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">3</td> <td style="border-left: 1px solid black; padding: 5px;">Min</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">2</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">4</td> <td style="border-left: 1px solid black; padding: 5px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">4</td> <td style="padding: 5px;">-1</td> <td style="padding: 5px;">2</td> <td style="border-left: 1px solid black; padding: 5px;">②</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px;">Max</td> <td style="padding: 5px;">5</td> <td style="padding: 5px;">5</td> <td style="border-left: 1px solid black; padding: 5px;">④</td> </tr> </table>	5	1	3	Min	2	5	4	1	4	-1	2	②	Max	5	5	④	M1		
5	1	3	Min																	
2	5	4	1																	
4	-1	2	②																	
Max	5	5	④																	
	$2 \neq 4 \Rightarrow$ no stable solution	A1	2																	
(b)	$(5 \ 1 \ 3) > (4 \ -1 \ 2)$	E1	1																	
(c)(i)	<p>A chooses 1 p chooses 2 $1-p$</p> <p>\therefore gain $5p + 2(1-p) = 3p + 2$ $1p + 5(1-p) = 5 - 4p$ $3p + 4(1-p) = 4 - p$</p> 	M1																		
		A1																		
		M1																		
		A1																		
		A1																		
	Therefore $3p + 2 = 5 - 4p \Rightarrow p = \frac{3}{7}$	B1✓		choosing the middle value																
	Therefore A plays 1 with $\frac{3}{7}$ 2 with $\frac{4}{7}$	B1✓	7																	
(ii)	Therefore the value is $3 \times \frac{3}{7} + 2 = \frac{23}{7}$	B1	1																	
	Total		11																	
	TOTAL		75																	