A-LEVEL
Mathematics

MM2B – Mechanics 2B
Mark scheme

6360
June 2016

Version 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Do not allow Mis-Reads in this question.				
Q	**Solution**	**Mark**	**Total**	**Comment**
1 (a)	Initial KE is $\frac{1}{2} \times 0.3 \times 8^2$ $= 9.6$ J	**M1** **A1**	 **2**	M1: Correct expression for KE. A1: Correct KE. CAO.
(b) (i)	KE = Initial KE + loss in PE $= 9.6 + 0.3 \times g \times 5$ $= 9.6 + 14.7$ $= 24.3$ J	 **B1** **M1** **A1**	 **3**	Do NOT accept constant acceleration formulae methods for any part of this question B1: Correct PE calculation, seeing either 14.7 or $0.3 \times g \times 5$ M1: Adding their PE to their answer to (a). A1: Correct KE. CAO.
(b) (ii)	Speed of stone is $\sqrt{\dfrac{24.3}{\frac{1}{2} \times 0.3}}$ $= 12.7279..$ ms $^{-1}$ $= 12.7$ ms $^{-1}$	**M1** **A1**	 **2**	M1: square root of their (b)(i)/0.15. OE. A1: Correct speed. CAO. Accept $9\sqrt{2}$
(b) (iii)	Either Stone is a particle Or No air resistance Or No resistance forces Or No wind	**E1**	**1**	Do not accept g is constant Do not accept No other forces acting
	Total		**8**	

Do not allow Mis-Reads in this question.				
Q	**Solution**	**Mark**	**Total**	**Comment**
2 (a)	$\mathbf{a} = (8 - 4t^3)\ \mathbf{i} - \ 18e^{-3t}\ \mathbf{j}$	**M1A1**	**2**	M1: Either term correct. A1: Correct acceleration. CAO.
(b) (i)	$\mathbf{F} = 2\mathbf{a}$ $= (16 - 8t^3)\ \mathbf{i} - \ 36e^{-3t}\ \mathbf{j}$	**M1** **A1**	 **2**	M1: 2× acceleration from (a) provided it is a vector. A1: Correct **F**. CAO
(ii)	When $t = 1$, $\mathbf{F} = 8\ \mathbf{i} - \ 36e^{-3}\ \mathbf{j}$ Magnitude is $\sqrt{(64 + [36e^{-3}]^2)}$ $= 8.1983..$ $= 8.20$ N	 **M1** **m1** **A1**	 **3**	 M1: Substituting $t = 1$ into their expression for **F**, must be a vector. m1: Finding the magnitude of their **F**, must square, add and square root. A1: Correct magnitude. CAO. Condone 8.2 if 8.198..seen
(c)	When **F** acts due south, east component is zero $16 - 8t^3 = 0$ $t = \sqrt[3]{2}$ $= 1.26$	 **M1** **A1**	 **2**	 M1: Setting i component equal to zero A1: Correct time. CAO. Accept $\sqrt[3]{2}$ or 1.259..
(d)	$\mathbf{r} = (4t^2 - \frac{1}{5}t^5)\ \mathbf{i} - 2e^{-3t}\ \mathbf{j} + \mathbf{c}$ When t = 0, $\mathbf{r} = 3\ \mathbf{i} - 5\ \mathbf{j}$, $\therefore \mathbf{c} = 3\ \mathbf{i} - 3\ \mathbf{j}$ $\therefore\ \mathbf{r} = (4t^2 - \frac{1}{5}t^5 + 3)\ \mathbf{i} - (3 + 2e^{-3t})\ \mathbf{j}$	**M1** **A1** **B1** **A1**	 **4**	M1: One component correct. Condone missing **c**. A1: Both components correct, condone missing **c**. B1: Correct constant. CAO. A1: Correct position vector and must be in the form a**i** +b**j**. CAO.
	Total		**13**	

Do not allow Mis-Reads in this question.

Q	Solution	Mark	Total	Comment
3 (a)	Symmetry	**E1**	**1**	
(b)	Moments about *AF* $400 \times 10 + 150 \times 12.5 + 400 \times 10 = 950\,d$	**M1** **M1A1**		M1: For 950 × distance of centre of mass (eg d or $\bar{x}$). M1: Any two correct terms on LHS. (OE) A1: Correct equation.
	$9875 = 950\,d$ $d = 10.39$ [or 10.39...] $= 10.4$ cm	**A1**	**4**	A1: Correct distance to 3sf. Condone 0.104 metres or 104 mm. Do not accept $\frac{395}{38}$
(c)	If θ is the angle required $\tan\theta = \frac{10.39}{35}$	**M1ft** **A1ft**		M1 ft: Seeing $\tan\theta = \dfrac{10.39}{35}$ or $\tan\theta = \dfrac{35}{10.39}$ [allow their 10.39] A1 ft: Correct expression for $\tan\theta$.
	$\theta = 16.541..$ $= 16.5°$	**A1**	**3**	A1: Correct angle to 3 or more sf. Condone use of 10.4 giving 16.5489 SC2 for 73.5°.
(d)	When it has been assumed that the centre of mass of each of the rectangles used is at its centre.	**E1**	**1**	**Or** Relating area to mass [do not accept mass distributed evenly]
	Total		**9**	

Do not allow Mis-Reads in this question.

Q	Solution	Mark	Total	Comment
4 (a)	Q is at rest, or not moving Tension is $8g$ or 78.4 N	**B1**	**1**	B1: Correct tension. CAO.
(b)	Resolving vertically for P: $T\cos\theta = 6g$	**M1A1**		M1: Resolve vertically. Condone use of $\sin\theta$. A1: Correct equation.
	$\cos\theta = \frac{6g}{8g}$ or $\frac{3}{4}$ $\theta = 41.4°$	**A1**	**3**	A1: Correct angle to 3 or more sf. Accept 0.723 radians
(c)	Resolving horizontally for P $T\sin\theta = m\frac{v^2}{r}$	**M1** **M1**		M1: $T\sin\theta$ seen, not $T\cos\theta$ M1: $m\frac{v^2}{r}$ seen
	$78.4 \times \frac{\sqrt{7}}{4} = 6\frac{5^2}{r} = \frac{150}{r}$	**A1**		A1: Correct equation with all values substituted.
	Radius = 2.8925 [or 2.8925...] = 2.89 metres	**A1**	**4**	A1: Correct distance to 3 or more sf. Condone 2.90 if 2.892..seen
	Total		**8**	

Do not allow Mis-Reads in this question.

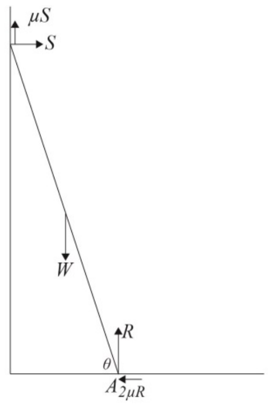
Q	Solution	Mark	Total	Comment
5 (a)	Conservation of energy $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgl(1 - \cos 30)$ $u^2 = v^2 + 2gl(1 - \cos 30)$ $v^2 = u^2 - 2gl(1 - \cos 30)$ $v = \sqrt{u^2 - 2gl(1 - \cos 30)}$	**M1** **A1** **A1**	 **3**	M1: Two correct KE terms with any *mgh* and *h* involves sin/cos 30/60. A1: Correct equation. A1: Correct expression oe. CAO. Accept $v = \sqrt{u^2 - 0.268gl}$
(b)	Resolving towards centre $T - mg\cos 30 = m\frac{v^2}{l}$ $T - mg\cos 30 = \frac{m}{l}[u^2 - 2gl(1 - \cos 30)]$ $T = \frac{mu^2}{l} + mg\cos 30 - 2mg(1 - \cos 30)$ $= \frac{mu^2}{l} + mg[3\cos 30 - 2]$	 **M1A1** **A1**	 **3**	 M1: Correct terms with any signs. A1: Correct equation. A1: Correct tension. CAO Accept $T = \frac{mu^2}{l} + 0.598mg$ Do not accept two unconnected terms in *mg*
(c)	Resolving vertically at *R* $T - mg = m\frac{u^2}{l}$ $T = mg + m\frac{u^2}{l}$	 **M1** **A1**	 **2**	 M1: Correct terms with any signs. Do not accept v^2 A1: Correct tension. CAO.
(d)	At highest point velocity is *V*, for string to remain taut, $m\frac{V^2}{l} > mg$ $V^2 > gl$ $\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - 2mgl$ $u^2 = V^2 + 4gl$ $u^2 > 5gl$ Minimum value of *u* is $\sqrt{5gl}$	 **M1** **A1** **m1** **A1**	 **4**	**Or** use (b) with 180 replacing 30 M1: Using $T > 0$. A1: Correct simplification. m1: Correct equation from conservation of energy. A1: Correct conclusion. No marks for $\sqrt{4gl}$

	Or $T = \frac{mu^2}{l} + (3\cos 180 - 2)mg$ $T = \frac{mu^2}{l} - 5mg$ $T > 0$ $u^2 > 5gl$ Minimum value of u is $\sqrt{5gl}$	**(M1)** **(A1)** **(m1)** **(A1)**	**(4)**	M1: Use (b) with 180 replacing 30. A1: Simplified with 5*mg*. m1: Using $T > 0$ A1: Correct conclusion.
	Total		**12**	

Do not allow Mis-Reads in this question.

Q	Solution	Mark	Total	Comment
6 (a)	Using $F = ma$, $m\frac{dv}{dt} = mg - \lambda mv$	**M1**		M1: Correct terms with any signs. Must see m in every term.
	$\therefore \frac{dv}{dt} = g - \lambda v$ **AG**	**A1**	**2**	A1: Correct result from correct working.
(b)	$\int \frac{dv}{g-\lambda v} = \int dt$	**M1**		M1: Correct separation of variables and forming two correct integrals.
	$-\frac{1}{\lambda}\ln(g-\lambda v) = t + c$	**M1A1**		M1: Either integration correct. A1: Both integrations correct with + c.
	When $t = 0$, $v = u$ $\Rightarrow c = -\frac{1}{\lambda}\ln(g - \lambda u)$	**A1**		A1: Correct constant. OE.
	$\ln(g - \lambda v) = -\lambda t + \ln(g - \lambda u)$ $\frac{g - \lambda v}{g - \lambda u} = e^{-\lambda t}$	**m1**		m1: Correctly eliminating ln.
	$g - \lambda v = (g - \lambda u)e^{-\lambda t}$ $v = \frac{1}{\lambda}(g - [g - \lambda u]e^{-\lambda t})$	**A1**	**6**	A1: Correct expression for v. OE
	Total		**8**	

Do not allow Mis-Reads in this question.

Q	Solution	Mark	Total	Comment
7 (a)		**B2**	**2**	B1: For 4 forces correct. B2: All forces correct. Forces must have arrow heads and appropriate labels. Friction forces do not need to contain μ, eg $F_{A.}$
(b)	Moments about A $lW\cos\theta = 2l\,S\sin\theta + 2l\,\mu\,S\cos\theta$	**M1A1**		M1: Moments about either end with 3 terms at least two of which are correct. Friction forces do not need to contain μ, eg $F_{A.}$ The three terms must either include length in all the terms or not in any of the terms Condone [cancelling l] $W\cos\theta = 2\,S\sin\theta + 2\,\mu\,S\cos\theta$ A1: Correct moment equation about either end.
	$\tan\theta = \frac{W-2\mu S}{2S}$			
	Resolve horizontally $2\mu\,R = S$ Resolve vertically $\mu\,S + R = W$	**B1** **B1**		B1: Resolve horizontally correctly B1: Resolve vertically correctly. Both do not need μ could use F
	$2\mu^2\,S + S = 2\,\mu\,W$ $S = \frac{2\mu W}{2\mu^2+1}$	**B 1**		A1: Correct reaction force. Note: $R = \frac{W}{2\mu^2+1}$ Could be $W = (2\mu^2+1)R$ Or $W = \frac{(2\mu^2+1)}{2\mu}S$
	$\tan\theta = \frac{W-2\mu\frac{2\mu W}{2\mu^2+1}}{2\frac{2\mu W}{2\mu^2+1}}$	**m1**		m1: Unsimplified expression for $\tan\theta$ in terms of μ (and W or R or S).
	$= \frac{2\mu^2+1-4\mu^2}{4\mu}$ $= \frac{1-2\mu^2}{4\mu}$	**A1**	**7**	A1: Correct answer.

(b)	**OR** Moments about the centre: $Rl\cos\theta = Sl\sin\theta + 2\mu Rl\sin\theta + \mu Sl\cos\theta$	**(M1)** **(A1)** **(A1)**		M1: Moments about centre with four terms at least two terms correct. Friction forces do not need to contain μ, eg $F_{A.}$ Condone [cancelling l] $R\cos\theta = S\sin\theta + 2\mu R\sin\theta + \mu S\cos\theta$ A1: Moment equation with correct terms but allow sign errors. A1: Correct moment equation about the centre.
	$S = 2\mu R$	**(B1)**		B1: Resolve horizontally correctly
	$R\cos\theta = 2\mu R\sin\theta + 2\mu R\sin\theta + 2\mu^2 R\cos\theta$	**(M1)**		M1: Substituting for S [or for R]
	$\cos\theta = 4\mu\sin\theta + 2\mu^2\cos\theta$ $4\mu\sin\theta = (1-2\mu^2)\cos\theta$	**(m1)**		m1: Eliminating R [or S].
	$\tan\theta = \dfrac{1-2\mu^2}{4\mu}$	**(A1)**	**(7)**	A1: Correct answer.
	Total		**9**	

Do not allow Mis-Reads in this question.				
Q	**Solution**	**Mark**	**Total**	**Comment**
8	Normal reaction on particle is 5 g cos 30 Frictional force is 5 g cos 30 × μ = 2$g$cos30 = $\sqrt{3}\ g$	**M1** **A1**		M1: R as 5$g$cos30 or 5$g$sin30 A1: Correct friction force, possibly in terms of μ. Accept any correct equivalents. [16.97 or 17.0]
	Initial EPE in string PR is $\frac{120\times5^2}{2\times6}$ = 250 J	**B1**		B1: Correct initial EPE.
	If particle moves a distance x when it is next at rest $\sqrt{3}\ g \times x + \frac{120\times(5-x)^2}{2\times6} + \frac{160\times x^2}{2\times4}$ = 5 g sin 30 × x + 250	**M1A1** **A1**		M1: Needs at least 4 terms [at least 3 correct with any signs] from: Work done [friction], EPE [in PQ], EPE [in PR], change in PE and initial EPE. See alternatives below for different x used; mark with version of x giving maximum mark A1: 5 terms correct with any signs. A1: 5 terms correct with correct signs.
	$\sqrt{3}\ g \times x + 10(5-x)^2 + 20x^2$ $= \frac{5}{2} g x + 250$ $30x^2 - 107.53x = 0$	**A1**		A1: Correct simplified quadratic. [see alternatives below] Condone rounding eg $30x^2 - 108x = 0$
	x = 0 or 3.5843 [or 3.5842] Particle moves 3.58 m Distance from Q is 7.58 m	**A1**	**8**	A1: Correct distance to 3 sf. Only accept 7.58.
	Total		**8**	

If x is taken as distance from Q equation becomes

$\sqrt{3}$ g (x – 4) + $\frac{120(9-x)^2}{2\times6}$ + $\frac{160(x-4)^2}{2\times4}$ = 5g sin 30 (x-4) + 250

Leads directly to 30 x^2 - 347.53 x + 910.12 = 0 and hence 7.58

If x is taken as distance from P equation becomes

$\sqrt{3}$ g (11-x) + $\frac{120(x-6)^2}{2\times6}$ + $\frac{160(11-x)^2}{2\times4}$ = 5g sin 30 (11 – x) + 250

Leading to 30x^2 - 552.47 x + 2447.17 = 0 and hence 7.42. Thus distance from Q is 7.58