Please write clearly in block capitals.

Centre number						Candidate number				

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS
Unit Decision 2

Wednesday 29 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

PB/Jun16/E3

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** below shows an activity diagram for a project. Each activity requires one worker. The duration required for each activity is given in hours.

(a) Find the earliest start time and the latest finish time for each activity and insert these values on **Figure 1**.

[4 marks]

(b) (i) Find the critical path.

[1 mark]

(ii) Find the float time of activity F.

[1 mark]

(c) Using **Figure 2** on page 3, draw a resource histogram to illustrate how the project can be completed in the minimum time, assuming that each activity is to start as early as possible.

[3 marks]

(d) (i) Given that there are two workers available for the project, find the minimum completion time for the project.

[2 marks]

(ii) Write down an allocation of tasks to the two workers that corresponds to your answer in part **(d)(i)**.

[1 mark]

Answer space for question 1

Figure 1

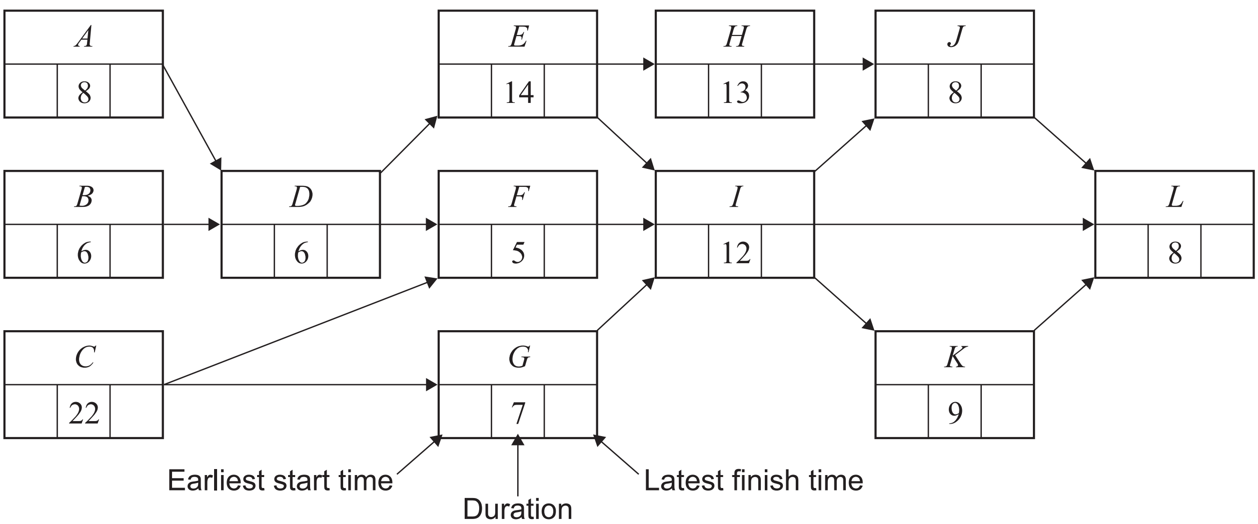

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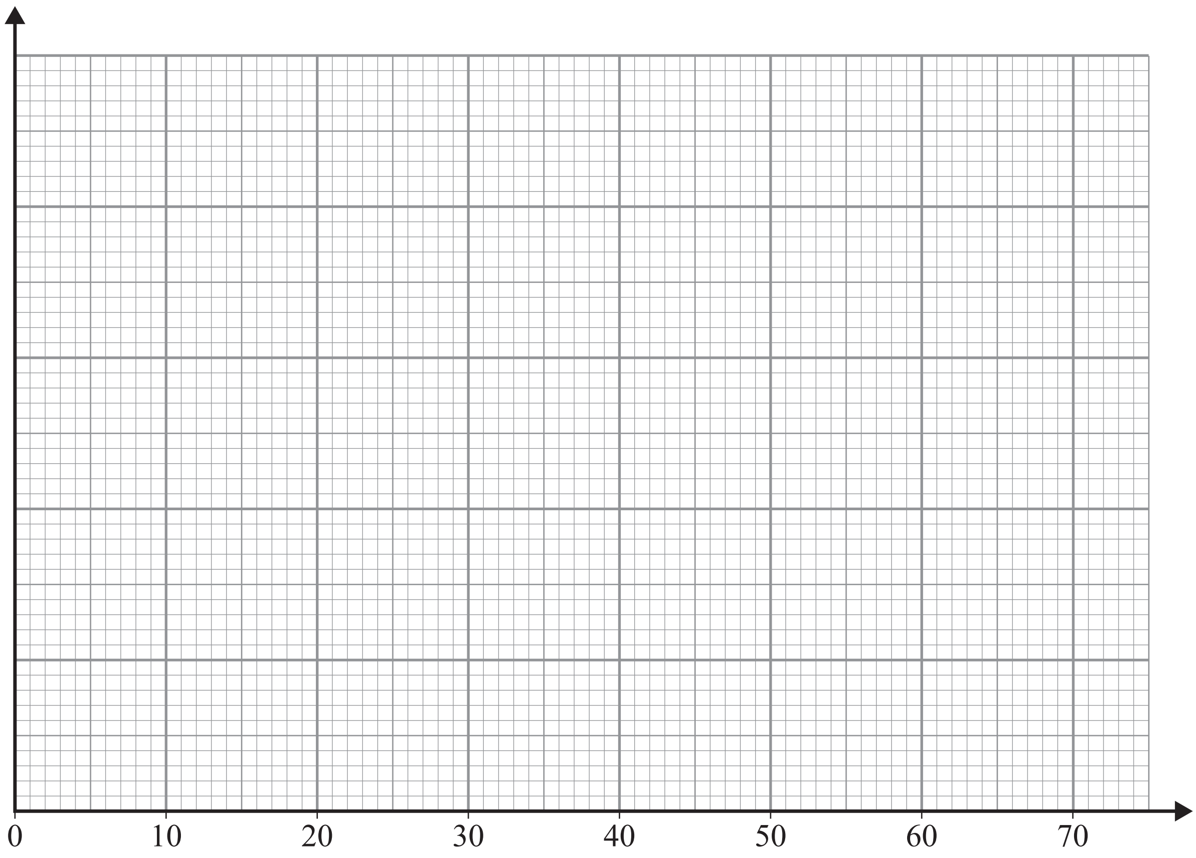
Figure 2



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2 Alan, Beth, Callum, Diane and Ethan work for a restaurant chain. The costs, in pounds, for the five people to travel to each of five different restaurants are recorded in the table below.

Alan cannot travel to restaurant 1 and Beth cannot travel to restaurants 3 and 5, as indicated by the asterisks in the table.

	Alan	**Beth**	**Callum**	**Diane**	**Ethan**
Restaurant 1	***	10	15	14	11
Restaurant 2	9	8	13	11	10
Restaurant 3	17	***	20	18	16
Restaurant 4	8	9	12	11	10
Restaurant 5	12	***	16	15	15

(a) Using the Hungarian algorithm, find all possible ways of allocating the five people each to a different restaurant, so that the total of their travelling costs is minimised.

[9 marks]

(b) Find the minimum total cost.

[1 mark]

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3

Maximise $P = 2x - 3y + 4z$

subject to

$$x + 2y + z \leqslant 20$$
$$x - y + 3z \leqslant 24$$
$$3x - 2y + 2z \leqslant 30$$

and $x \geqslant 0\ ,\ \ y \geqslant 0\ ,\ \ z \geqslant 0\ .$

(a) Display the linear programming problem in a Simplex tableau.

[2 marks]

(b) (i) The first pivot to be chosen is from the z-column. Identify the pivot and explain why this particular value is chosen.

[2 marks]

(ii) Perform one iteration of the Simplex method.

[3 marks]

(iii) Perform one further iteration.

[3 marks]

(c) Interpret your final tableau and state the values of your slack variables.

[3 marks]

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4 Monica and Vladimir play a zero-sum game. The game is represented by the following pay-off matrix for Monica.

Vladimir

	Strategy	**D**	**E**	**F**
	A	-1	-5	0
Monica	**B**	-4	1	-5
	C	-2	4	-3

(a) Explain what is meant by the term 'zero-sum' game.

[1 mark]

(b) Determine the play-safe strategy for each player.

[2 marks]

(c) Find the optimal mixed strategy for Monica and show that the value of the game is -1.4 .

[8 marks]

(d) Find the optimal mixed strategy for Vladimir.

[4 marks]

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5 Robert is planning to renovate four houses, A, B, C and D, at the rate of one per month. The houses can be renovated in any order but the costs will vary because some of the materials left over from renovating one house can be used for the next one. The expected profits, in hundreds of pounds, are given in the table below.

Month	Houses already renovated	Expected profit (£00s)			
		A	B	C	D
March	–	40	55	60	70
April	A	–	60	71	75
	B	50	–	70	77
	C	47	56	–	79
	D	52	68	68	–
May	A and B	–	–	75	81
	A and C	–	59	–	80
	A and D	–	62	74	–
	B and C	56	–	–	85
	B and D	59	–	77	–
	C and D	57	60	–	–
June	A, B and C	–	–	–	88
	A, B and D	–	–	83	–
	A, C and D	–	70	–	–
	B, C and D	66	–	–	–

(a) By completing the table opposite, use dynamic programming, working backwards from June, to find the schedule that maximises the total expected profit.

[10 marks]

(b) State this maximum expected profit.

[1 mark]

Stage (Month)	State (houses renovated)	Action	Calculation	Profit (£00s)
June	A, B, C	D		88
	A, B, D	C		83
	A, C, D	B		70
	B, C, D	A		66
May	A, B	C	75 + 88	163

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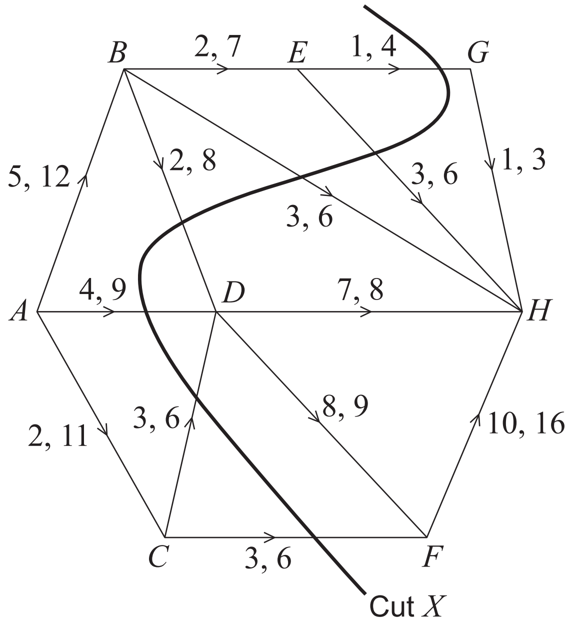
6 The network shows a system of pipes with lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut X.

[1 mark]

(ii) Hence state what can be deduced about the maximum flow from A to H.

[1 mark]

(b) **Figure 3** shows a partially completed diagram for a feasible flow of 28 litres per second from A to H.

Indicate, on **Figure 3**, the flows along the edges BD, BE and CD.

[3 marks]

(c) (i) Using your feasible flow from part **(b)** as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4**.

[3 marks]

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from A to H. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network.

[4 marks]

(iii) State the maximum flow and indicate a maximum flow on **Figure 5**.

[2 marks]

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Figure 3

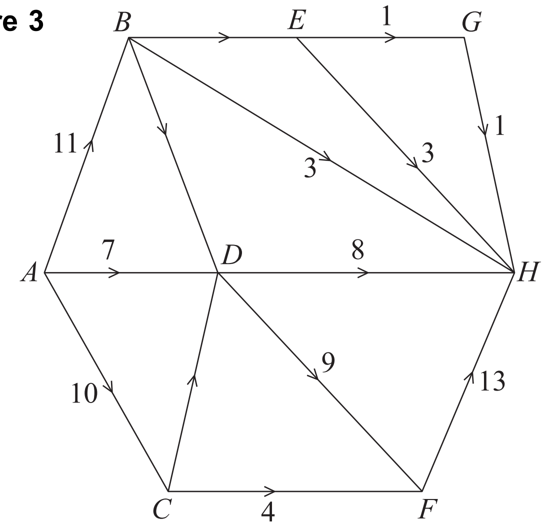
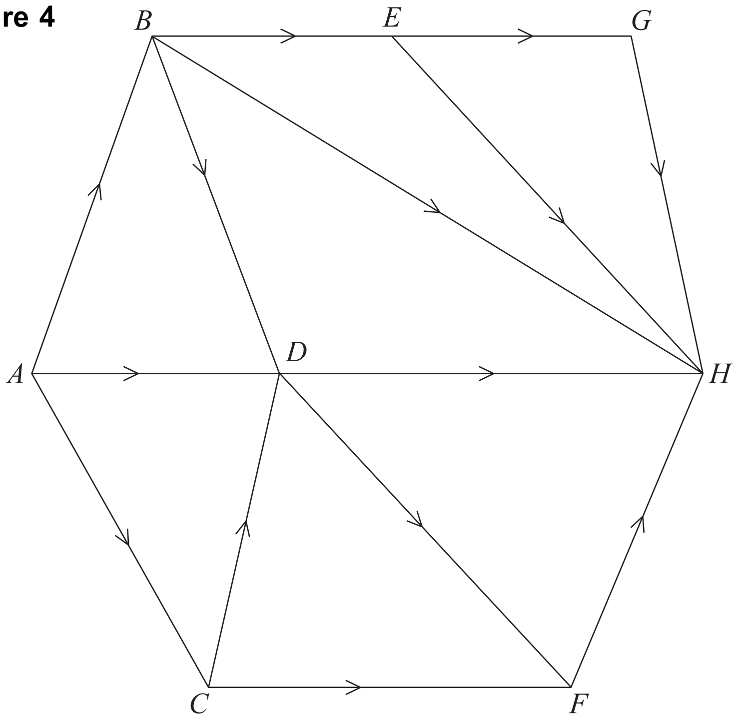
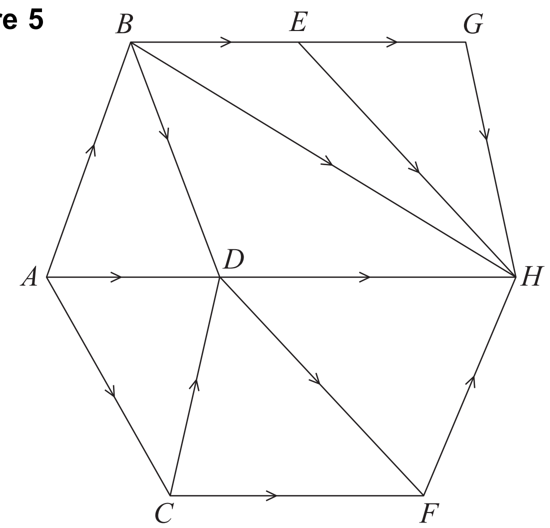


Figure 4



Flow	Value

Figure 5



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END OF QUESTIONS