AQA

A-LEVEL

Mathematics

Pure Core 2 – MPC2
Mark scheme

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
	(Area of sector =) $\frac{1}{2}r^2\theta$	**M1**		$\frac{1}{2}r^2\theta$ seen, or used, for the sector area
	$\frac{1}{2}(5^2)\theta = 15 \quad \left(\theta = \frac{15}{12.5}\right)$	**A1**		A correct equation in θ or in $r\theta$ eg $2.5r\theta = 15$
	(Perimeter of sector =) 5 + 5 + 5θ	**M1**		$r + r + r\theta$ seen, or used, for the perimeter
	$= 10 + 5\times\frac{6}{5} = 16$ (cm)	**A1**	**4**	16
	Total		**4**	

Q2	Solution	Mark	Total	Comment
(a)	$\frac{AC}{\sin 48^\circ} = \frac{20}{\sin 72^\circ}$	**M1**		Correct use of sine rule with *AC* being the only unknown
	$AC = \frac{20\sin 48^\circ}{\sin 72^\circ} \quad (= \frac{14.86...}{0.951...})$	**A1**		Correct expression for *AC*. PI by 15.62(7774..)
	= 15.62(7774..) = 15.6 (cm to 3 sf)	**A1**	**3**	AG Need some intermediate evaluation between $\frac{20\sin 48^\circ}{\sin 72^\circ}$ and 15.6
(b)	Angle *ACB* = 60º	**B1**		Either *ACB* = 60º stated or used or seen on diagram or *AB* = AWRT 18.2
	$(AM^2=)10^2 + (15.6)^2 - 2\times10\times15.6\times\cos C$	**M1**		RHS of relevant cosine rule used correctly
	$=10^2 + (15.6)^2 - 156$	**m1**		$10^2 + (15.6)^2 - 156$ OE; accept evaluation to, 187 to 188 incl., as evidence
	AM = 13.7 (cm to 3 sf)	**A1**	**4**	Condone more accurate answer
	Total		**7**	

(b) Allow use of 15.6 or better for *AC*

(b) **Altn using perpendicular from *A* to *BC***
Either *ACB* = 60º stated or used or seen on diagram or *AB* = AWRT 18.2 (**B1**)
$(AM^2=)\ (15.6\sin 60)^2 + (10 - 15.6\cos 60)^2$ OR $(AM^2=)\ (18.2\sin 48)^2 + (18.2\cos 48 - 10)^2$ (**M1**)
$= (13.5)^2+(2.2)^2$ (**m1**) Correct evaluations to at least 1dp accept evaluation to, 187 to 188 incl., as evidence.
AM = 13.7 (cm to 3 sf) (**A1**) Condone more accurate answer

Q3	Solution	Mark	Total	Comment
(a)	(3rd term=) $ar^2 = 48(0.6)^2$	M1		ar^{3-1} stated or used
	$= 17.28$	A1	2	OE fraction eg 432/25. NMS 17.28 OE scores 2 marks unless FIW.
(b)	$\{S_\infty =\}\ \dfrac{a}{1-r} = \dfrac{48}{1-0.6}$	M1		$\dfrac{a}{1-r}$ <u>used</u> with a = 48 and r = 0.6 OE
	$\{S_\infty =\}$ 120	A1	2	Correct exact value for S_∞. NMS 120 scores 2 marks unless FIW.
(c)	$\sum_{n=4}^{\infty} u_n = S_\infty - \sum_{n=1}^{3} u_n$	M1		OE eg RHS = $S_\infty - \left(a + ar + ar^2\right)$
	$\sum_{n=1}^{3} u_n$ = (48+28.8 + c's (**a**))	A1F		OE eg $\sum_{n=1}^{3} u_n = \dfrac{48\left(1-0.6^3\right)}{1-0.6}$ (=94.08) PI
	$\sum_{n=4}^{\infty} u_n$ = 120 − 94.08 = 25.92	A1	3	25.92 OE exact value
	Altn.			
	$\sum_{n=4}^{\infty} u_n = \dfrac{u_4}{1-r}$	(M1)		
	$u_4 = 17.28 \times 0.6 = 10.368$	(A1F)		Ft on c's **(a)**×0.6. PI by $\sum_{n=4}^{\infty} u_n$ = correct evaluation of 1.5×c's**(a)**
	$\sum_{n=4}^{\infty} u_n = \dfrac{10.368}{1-0.6}$ = 25.92	(A1)		25.92 OE exact value
	Total		**7**	

Q4	Solution	Mark	Total	Comment
(a)	$\frac{2}{x^2} = 2x^{-2}$	B1		PI by its derivative as $-4x^{-3}$ or $4x^{-3}$
	$\frac{d^2y}{dx^2} = -4x^{-3} - \frac{1}{4}$	M1 A1	3	Differentiating one term correctly. ACF
(b)(i)	$\frac{2}{x^2} - \frac{x}{4} = 0$	M1		
	$(x_M =)\ \ 2$	A1	2	**NMS** 2/2 for correct answer.
(b)(ii)	(At M) $\frac{d^2y}{dx^2} = -\frac{4}{8} - \frac{1}{4} < 0$, so max.	E1	1	Using c's x_M and c's $\frac{d^2y}{dx^2}$ to show $\frac{d^2y}{dx^2}$ is negative and stating conclusion ie max.
(b)(iii)	$\int\left(\frac{2}{x^2} - \frac{x}{4}\right)dx = -2x^{-1} - \frac{x^2}{8} (+c)$	M1		Attempt to integrate $\frac{dy}{dx}$ with at least one of the two terms integrated correctly.
	$(y =)\ -2x^{-1} - \frac{x^2}{8}\ (+c)$	A1		$-2x^{-1} - \frac{x^2}{8}$ OE ; condone unsimplified
	When $x = 2$, $y = 2.5 \Rightarrow 2.5 = -1 - 0.5 + c$	M1		Subst. x = c's **(b)**, $y = 2.5$ into $y = F(x)$+'c' in attempt to find constant of integration, where $F(x)$ follows attempted integration of expression for $\frac{dy}{dx}$
	$y = -2x^{-1} - \frac{x^2}{8} + 4$	A1	4	ACF but with signs and coeffs simplified
	Total		**10**	

Q5	Solution	Mark	Total	Comment
(a)	$132 = 160p + q$	M1		Seen or used
	$20 = 20p + q$	M1		Seen or used
	$112 = 140p$	m1		Valid method to solve the correct two simultaneous eqns in p and q to at least the stage $112 = 140p$ OE or $28 = 7q$ OE PI by correct values for both p and q from two correct simultaneous equations
	$p = \frac{112}{140} \quad \left(= \frac{4}{5}\right)$	A1		ACF
	$q = 4$	A1	5	$q = 4$
(b)	$160 = \frac{4}{5}u_1 + 4 \qquad u_1 = 195$	B1F	1	Ft on $u_1 = \frac{160 - \text{c's } q}{\text{c's } p}$, provided u_1 is exact and p and q are both positive.
	Total		6	

Q6	Solution	Mark	Total	Comment
(a)	$\sin^{-1} 0.6 = 0.64(35...)$ $(= \beta)$	B1		PI by one correct value for x to at least 2dp or 2sf
	$x + 0.7 = \beta$, $x + 0.7 = \pi - \beta$ (=2.4(98..))	M1		$x + 0.7 = \beta$ and $x + 0.7 = \pi - \beta$ where β is the c's value for $\sin^{-1} 0.6$
	$x = -0.056$, 1.8 (to 2 sf)	A1	3	Must be correct 2sf values ie −0.056 , 1.8 Ignore any values outside given interval. **SC NMS** Condone>2sf and mark as B1 B1 max. {−0.056(498..); 1.7(9809..)}
(b)(i)	$5\cos^2\theta - \cos\theta = 1 - \cos^2\theta$	M1		Replacing $\sin^2\theta$ by $1 - \cos^2\theta$
	$6\cos^2\theta - \cos\theta - 1 = 0$	A1		
	$(2\cos\theta - 1)(3\cos\theta + 1)$ (= 0)	m1		$(2\cos\theta \pm 1)(3\cos\theta \pm 1)$ PI by the two 'correct' roots with correct/incorrect signs
	(Possible values of $\cos\theta$ =) $\frac{1}{2}$, $-\frac{1}{3}$	A1	4	The two correct values of $\cos\theta$.
(b)(ii)	When $\cos\theta = -\frac{1}{3}$, $\sin^2\theta = \frac{8}{9}$	B1		
	$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{(\pm)\sqrt{\frac{8}{9}}}{-\frac{1}{3}}$	M1		$\tan\theta = \frac{\sin\theta}{\cos\theta}$ **used**; could be used with either of c's values of $\cos\theta$ from **(b)(i)** and a corresponding value of $\sin\theta$
	So a (+'ve) value for $\tan\theta$ is $-\sqrt{\frac{8}{9}} \div \left(-\frac{1}{3}\right) = \sqrt{8} = 2\sqrt{2}$	A1	3	CSO A.G. Be convinced.
	Total		**10**	
(a)	Eg NMS $x = -0.06$, 1.80 scores B0B1			
(b)(ii) Alt	$\sec\theta = -3$, $\sec^2\theta = 9$ **(B1)**; $\tan^2\theta = \sec^2\theta - 1 = 9 - 1$ **(M1); (**+'ve) value of $\tan\theta$ is $\sqrt{8} = 2\sqrt{2}$ **(A1CSO)**			

Q7	Solution	Mark	Total	Comment
(a)(i)	Translation $\begin{bmatrix}0\\1\end{bmatrix}$	**E2,1,0**	**2**	E2: 'translat…' and $\begin{bmatrix}0\\1\end{bmatrix}$ OE. If not E2 award E1for 'translat… in *y*-dir' OE. More than one transformation scores 0/2
(a)(ii)	Stretch (**I**) in *x*-direction (**II**) scale factor 9 (**III**)	**M1** **A1**	 **2**	Need (**I**) **and** either (**II**) or (**III**) Need (**I**) and (**II**) and (**III**) More than one transformation scores 0/2
(b)(i)	$\int_0^9 \left(1+\sqrt{x}\right)\mathrm{d}x$ = 9 + 18 = 27	**B1**	**1**	27
(b)(ii)	h = 2.25	**B1**		h = 2.25 OE stated or used. (PI by *x*-values 0, 2.25, 4.5, 6.75, 9 provided no contradiction)
	f(x) = $4^{\frac{x}{9}}$ $\mathrm{I} \approx \frac{h}{2}$ {f(0)+f(9)+2[f(2.25)+f(4.5)+f(6.75)]}	**M1**		h/2{f(0)+f(9)+2[f(2.25)+f(4.5)+f(6.75)]} OE summing of areas of the 'trapezia'..
	$\frac{h}{2}$ with $\{\ldots\}=1+4+2\left(4^{\frac{1}{4}}+4^{\frac{1}{2}}+4^{\frac{3}{4}}\right)$ $= 5+2\left(\sqrt{2}+2+2\sqrt{2}\right) = 9+6\sqrt{2}$ $(\mathrm{I} \approx \frac{2.25}{2}[9+8.48...]=1.125\times 17.485..)$	**A1**		OE Accept 2sf or better evidence for surds. Can be implied by later <u>correct</u> work provided >1 term or a single term which rounds to 19.7
	(= 19.67…) = 19.7 (to 1 dp)	**A1**	**4**	CAO Must be 19.7 **SC** 5strips used: **Max** B0M1A0, 19.6 A1
(b)(iii)	Area of shaded region ≈ $\int_0^9 \left(1+\sqrt{x}\right)\mathrm{d}x - \int_0^9 4^{\frac{x}{9}}\,\mathrm{d}x$	**M1**		
	= 27 − 19.7 = 7.3	**A1F**		Ft on [c's **(b)(i)** − c's **(b)(ii)**] provided this gives a value>0.
	Since trapezia cover larger area than area under lower curve, 19.7 is overestimate so subtracting this from the true area, 27, under upper curve will lead to an **underestimate** of the true area of shaded region.	**E1**	**3**	Need both the final answer '**underestimate**' plus mention of the fact that the trapezium rule gives overestimate as trapezia cover larger area-cand could show this on a diagram. (E1 is dep on M1 but not on the A1F)
	Total		**12**	
(a)(i)	Example: 'translating 1 in positive *y*' OE (**E2**)			
(b)(ii)	For guidance, separate trap. 2.71(5..) + 3.84(0..)+5.43(1..)+7.68(1..). NB 3/4 possible if values to 2sf			
(b)(ii)	MR of f(x), but <u>**NOT**</u> from an attempted integration, <u>**max**</u> **B1M1A0A0**			

Q8	Solution	Mark	Total	Comment
	Gradient of the line $3y-2x=1$ is $\frac{2}{3}$	**B1**		(Gradient) $\frac{2}{3}$ seen or used. Condone 0.66, 0.67 or better for $\frac{2}{3}$.
	$\frac{dy}{dx} = \frac{1}{2}x^{-0.5}$	**B1**		Correct differentiation of $x^{\frac{1}{2}}$
	At A, $\frac{1}{2}x^{-0.5} = \frac{2}{3}$	**M1**		c's $\frac{dy}{dx}$ expression = c's numerical gradient of given line.
	$A\left(\frac{9}{16}, \frac{3}{4}\right)$	**A1**		Correct exact coordinates of A
	Eqn of tang at A: $y-\frac{3}{4}=\frac{2}{3}\left(x-\frac{9}{16}\right)$	**A1**	5	ACF eg $y=\frac{2}{3}x+\frac{3}{8}$ or eg $3y-2x=\frac{9}{8}$ must be exact
	Total		**5**	
Examples	Cand. writes $0.5x^{-0.5}=k$, and stops, where $k = -\frac{2}{3}$ or 2 or –2. Mark these types as (**B0, B1, M1A0A0**)			

Q9	Solution	Mark	Total	Comment
(a)	$3x\log 2 = \log 5$	M1		OE eg $3x = \log_2 5$ or eg $x\log 8 = \log 5$
	$x = 0.773(976...)$ = 0.774 (to 3sf)	A1	2	Condone > 3sf. If use of logarithms not explicitly seen then score 0/2
(b)	$\log_a \frac{k}{2} = \frac{2}{3}$	M1		Either $\log k - \log 2 = \log\frac{k}{2}$ or $\frac{2}{3} = \log a^{\frac{2}{3}}$ seen at any stage
	$\frac{k}{2} = a^{\frac{2}{3}}$	A1		OE eqn with logs eliminated with no incorrect work
	$a^{\frac{2}{3}} = \frac{k}{2} \Rightarrow a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$	m1		$a^{\frac{m}{n}} = C \Rightarrow a = C^{\frac{n}{m}}$
		A1	4	$a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$ OE exact form with no obvious incorrect working
(c)(i)	$(1+2x)^3 = 1 + 3(2x) + 3(2x)^2 + (2x)^3$ $= 1 + 6x + 12x^2 + 8x^3$	B3,2,1	3	B3: expansion correct and simplified B2: 3 of the 4 terms correct and simplified B2; 4 terms correct but not all simplified B1 2 of the 4 terms correct and simplified (ignore the ordering of the terms)
(c)(ii)	$[(1+2n)^3 - 8n] = 1 - 2n + 12n^2 + 8n^3$	B1F		Ft at most two incorrect coefficients in **(c)(i)**
	$\log(1+2n) + \log 4(1+n^2) = \log 4(1+n^2)(1+2n)$	M1		Log law 1 applied correctly to RHS of given eqn., ignore base. Those who rearrange the terms first before applying log law 2 correctly must also attempt to deal with the resulting fraction in a correct manner.
	Given equation becomes $1 - 2n + 12n^2 + 8n^3 = 8n^3 + 4n^2 + 8n + 4$ $8n^2 - 10n - 3$ (=0)	A1		Correct three term quadratic
	$(4n+1)(2n-3)$ (= 0)	A1		PI by correct two roots from a correct quadratic equation
	$n = -\frac{1}{4}$, $n = \frac{3}{2}$	A1	5	Need both as the final two values of *n* with no extras
	Total		**14**	
(b)	Example: $\log k - \log 2 = \frac{\log k}{\log 2} = \frac{2}{3}$, $\frac{\log k}{\log 2} = \log a^{\frac{2}{3}}$ **(M1)**, $\frac{k}{2} = a^{\frac{2}{3}}$ **(A0)**, $a = \left(\frac{k}{2}\right)^{\frac{3}{2}}$ **(m1) (A0)**			