

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	



General Certificate of Education
Advanced Level Examination
June 2015

Mathematics

MFP3

Unit Further Pure 3

Wednesday 13 May 2015 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



J U N 1 5 M F P 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

1 It is given that $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where $f(x, y) = \frac{x + y^2}{x}$

and $y(2) = 5$

(a) Use the Euler formula

$$y_{r+1} = y_r + hf(x_r, y_r)$$

with $h = 0.05$, to obtain an approximation to $y(2.05)$.

[2 marks]

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$$

with your answer to part **(a)**, to obtain an approximation to $y(2.1)$, giving your answer to three significant figures.

[3 marks]

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2

By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} + (\tan x)y = \tan^3 x \sec x$$

given that $y = 2$ when $x = \frac{\pi}{3}$.

[9 marks]

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3 (a) (i) Write down the expansion of $\ln(1 + 2x)$ in ascending powers of x up to and including the term in x^4 .

[1 mark]

(ii) Hence, or otherwise, find the first two non-zero terms in the expansion of

$$\ln[(1 + 2x)(1 - 2x)]$$

in ascending powers of x and state the range of values of x for which the expansion is valid.

[3 marks]

(b) Find $\lim_{x \rightarrow 0} \left[\frac{3x - x\sqrt{9+x}}{\ln[(1+2x)(1-2x)]} \right]$.

[4 marks]

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4 (a) Explain why $\int_2^{\infty} (x - 2)e^{-2x} dx$ is an improper integral.

[1 mark]

(b) Evaluate $\int_2^{\infty} (x - 2)e^{-2x} dx$, showing the limiting process used.

[6 marks]

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5 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 36 \sin 3x$$

[7 marks]

(b) It is given that $y = f(x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 36 \sin 3x$$

such that $f(0) = 0$ and $f'(0) = 0$.

(i) Show that $f''(0) = 0$.

[1 mark]

(ii) Find the first two non-zero terms in the expansion, in ascending powers of x , of $f(x)$.

[3 marks]

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6 A differential equation is given by

$$4\sqrt{x^5} \frac{d^2y}{dx^2} + (2\sqrt{x})y = \sqrt{x}(\ln x)^2 + 5, \quad x > 0$$

(a) Show that the substitution $x = e^{2t}$ transforms this differential equation into

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = 4t^2 + 5e^{-t}$$

[7 marks]

(b) Hence find the general solution of the differential equation

$$4\sqrt{x^5} \frac{d^2y}{dx^2} + (2\sqrt{x})y = \sqrt{x}(\ln x)^2 + 5, \quad x > 0$$

[10 marks]

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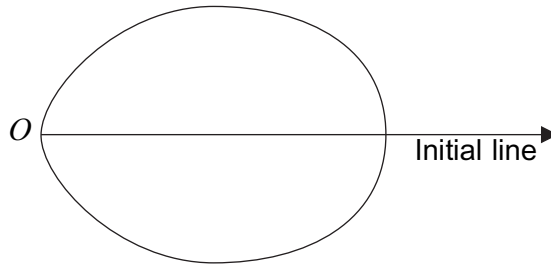
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7 The diagram shows the sketch of a curve C_1 .



The polar equation of the curve C_1 is

$$r = 1 + \cos 2\theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

(a) Find the area of the region bounded by the curve C_1 .

[5 marks]

(b) The curve C_2 whose polar equation is

$$r = 1 + \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

intersects the curve C_1 at the pole O and at the point A . The straight line drawn through A parallel to the initial line intersects C_1 again at the point B .

(i) Find the polar coordinates of A .

[4 marks]

(ii) Show that the length of OB is $\frac{1}{4}(\sqrt{13} + 1)$.

[6 marks]

(iii) Find the length of AB , giving your answer to three significant figures.

[3 marks]

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END OF QUESTIONS



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